MATHEMATICS
EXTENSION 2

SmarterEd Maths & Sciences

HSC REVISION SERIES 2026

Welcome to the SmarterMaths 2026 HSC Extension 2 Revision Series.

Key features of this year's Revision Series include:

- ~18 hours of carefully selected HSC revision questions
- Targeted practice across the most important HSC question types
- Designed for students aiming for a Band 5-6 result
- Higher weighting towards historically difficult question types
- Topic and question weightings reflect historical HSC exam allocations
- Build your own personalised revision resource by ***carefully reviewing and annotating*** these worksheets throughout Term 3.

**Philosophy: Every question has earned its place. Historical HSC trends, common question types and past areas of student difficulty have all influenced the selection process.*

IMPORTANT

If you've already completed some of these questions during the year, **that's a good thing.**

Many of the highest-performing students credit repeated practice with challenging past HSC questions as a key ingredient in their HSC success. This revision series focuses on the highest-value question types, helping you build confidence and critically, *speed* through the exam.

HSC Final Study – Extension 2: Vectors

Syllabus: V1 Vectors

Worksheet 2 of 2 | Estimated exam weighting: 18.2%

Key Areas addressed by this worksheet:

Vectors and Geometry (7.7%)

- *Vectors and Geometry* **(★)** is a 550-pound Silverback of the Extension 2 course that has been examined in the longer answer section of every new syllabus exam, peaking in the period 2023-24.
- These allocations need to be added to impressive contributions each year in the prior period 2020-2022. In short, calling it key revision for achieving a band 5-6 result is a statement of the bleeding obvious.
- Revision questions include multiple examples of both 2D and 3D shapes which have both featured in new syllabus exams.
- The most common question type looks at pyramids **(★)**, both square (2021) and triangular (2023, 2022).
- 2D quadrilaterals and triangles **(★)** have been the basis of many recent HSC questions and feature in the revision set.
- *2023 Ext2 15c* and *2021 Ext2 16a* look at spheres **(★)**. Both caused issues and are important revision examples.

KEY: ★ Priority Revision – content or specific questions flagged by SM analysis as key revision.

"Students should take wandering outdoor walks when studying Vectors and Geometry, so that the mind might be nourished and refreshed by the open air and deep breathing."

~ Seneca

EXTENSION 2 2026

HSC Revision Series

Smarter Ed

Vectors (2 of 2)

V1 Working With Vectors

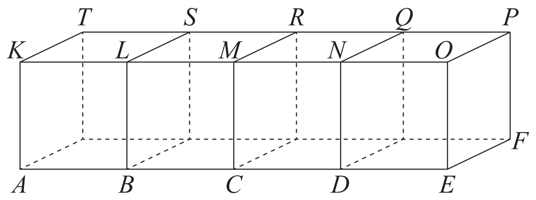
Vectors and Geometry

Exam Equivalent Time: 90 minutes *(based on allocation of 1.5 minutes per mark)*

Questions

1. Vectors, EXT2 V1 2021 HSC 1 MC

Four cubes are placed in a line as shown on the diagram.



Which of the following vectors is equal to $\overrightarrow{AB} + \overrightarrow{CQ}$

A. $\overrightarrow{AQ}$

B. $\overrightarrow{CP}$

C. $\overrightarrow{PB}$

D. $\overrightarrow{RA}$

2. Vectors, EXT2 V1 2022 HSC 9 MC

Let A and B be two distinct points in three-dimensional space. Let M be the midpoint of AB.

Let S_1 be the set of all points P such that $\overrightarrow{AP} \cdot \overrightarrow{BP} = 0$.

Let S_2 be the set of all points N such that $\left|\overrightarrow{AN}\right| = \left|\overrightarrow{MN}\right|$.

The intersection of S_1 and S_2 is the circle S.

What is the radius of the circle S?

A. $\dfrac{\left|\overrightarrow{AB}\right|}{2}$

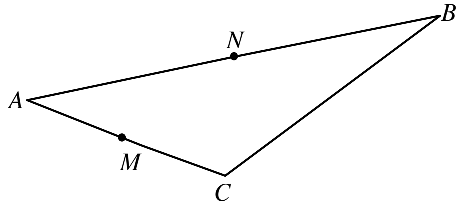
B. $\dfrac{\left|\overrightarrow{AB}\right|}{4}$

C. $\dfrac{\sqrt{3}\left|\overrightarrow{AB}\right|}{2}$

D. $\dfrac{\sqrt{3}\left|\overrightarrow{AB}\right|}{4}$

3. Vectors, EXT2 V1 EQ-Bank 14

In triangle ABC, M is the midpoint of AC and N is the midpoint of AB.



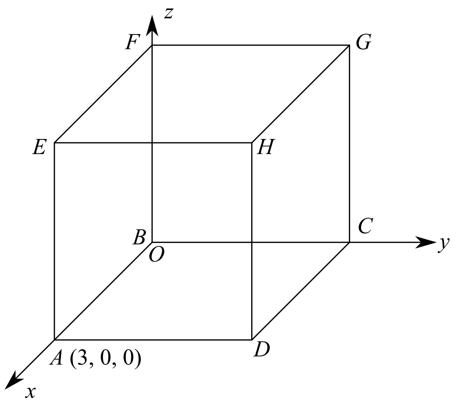
Use vector methods to prove that

a. $MN = \dfrac{1}{2}CB$ *(2 marks)*

b. MN is parallel to CB *(1 mark)*

4. Vectors, EXT2 V1 EQ-Bank 19

A cube with side length 3 units is pictured below.



a. Calculate the magnitude of vector $\overrightarrow{AG}$. *(1 mark)*

b. Find the acute angle between the diagonals $\overrightarrow{AG}$ and $\overrightarrow{BH}$. *(3 marks)*

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5. Vectors, EXT2 V1 2025 HSC 15a

The adjacent sides of a parallelogram are represented by the vectors $\underset{\sim}{a} = 4\underset{\sim}{i} + 3\underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k}$.

Show that the area of the parallelogram is $6\sqrt{10}$ square units. *(4 marks)*

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6. Vectors, EXT2 V1 EQ-Bank 22

Given $\underset{\sim}{a} = (3, -2, 1)$ and $\underset{\sim}{b} = (4, 3, -4)$, verify numerically that

$$\underset{\sim}{a} \cdot \underset{\sim}{b} = \left|\underset{\sim}{a}\right|\left|\underset{\sim}{b}\right|\cos\theta = x_1x_2 + y_1y_2 + z_1z_2$$

where $\underset{\sim}{a} = (x_1, y_1, z_1)$ and $\underset{\sim}{b} = (x_2, y_2, z_2)$ *(4 marks)*

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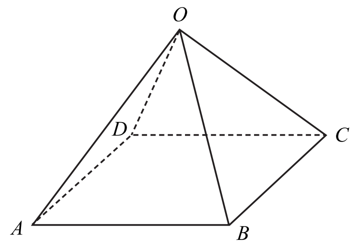
7. Vectors, EXT2 V1 EQ-Bank 25



Let $OABCD$ be a right square pyramid where $\underset{\sim}{a} = \overrightarrow{OA},\ \underset{\sim}{b} = \overrightarrow{OB},\ \underset{\sim}{c} = \overrightarrow{OC}$ and $\underset{\sim}{d} = \overrightarrow{OD}$.

Show that $\underset{\sim}{a} + \underset{\sim}{c} = \underset{\sim}{b} + \underset{\sim}{d}$. *(3 marks)*

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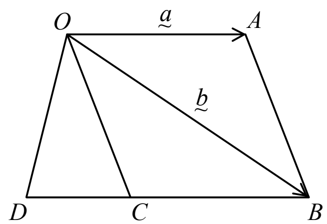
8. Vectors, EXT2 V1 EQ-Bank 27

$OABD$ is a trapezium in which $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$.

OC is parallel to AB and $DC : CB = 1 : 2$



Using vectors, express $\overrightarrow{DA}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. *(3 marks)*

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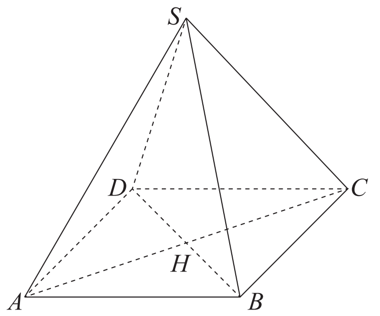
9. Vectors, EXT2 V1 2021 HSC 12e

The diagram shows the pyramid $ABCDS$ where $ABCD$ is a square. The diagonals of the square bisect each other at H.



i. Show that $\overrightarrow{HA} + \overrightarrow{HB} + \overrightarrow{HC} + \overrightarrow{HD} = \underset{\sim}{0}$ *(1 mark)*

Let G be the point such that $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} + \overrightarrow{GD} + \overrightarrow{GS} = \underset{\sim}{0}$.

ii. Using part (i), or otherwise, show that $4\overrightarrow{GH} + \overrightarrow{GS} = \underset{\sim}{0}$. *(2 marks)*

iii. Find the value of λ such that $\overrightarrow{HG} = \lambda\overrightarrow{HS}$ *(1 mark)*

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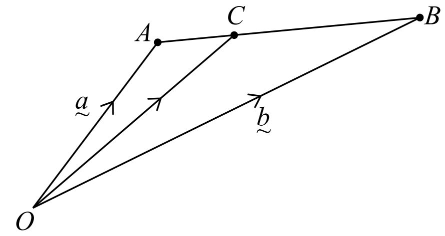
10. Vectors, EXT2 V1 2020 HSC 15b

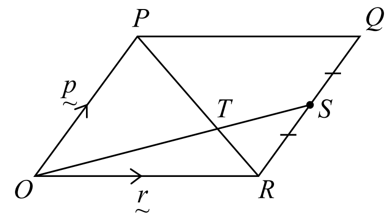
The point C divides the interval AB so that $\dfrac{CB}{AC} = \dfrac{m}{n}$. The position vectors of A and B are $\underset{\sim}{a}$ and $\underset{\sim}{b}$ respectively, as shown in the diagram.



i. Show that $\overrightarrow{AC} = \dfrac{n}{m+n}\left(\underset{\sim}{b} - \underset{\sim}{a}\right)$. *(2 marks)*

ii. Prove that $\overrightarrow{OC} = \dfrac{m}{m+n}\underset{\sim}{a} + \dfrac{n}{m+n}\underset{\sim}{b}$. *(1 mark)*

Let $OPQR$ be a parallelogram with $\overrightarrow{OP} = \underset{\sim}{p}$ and $\overrightarrow{OR} = \underset{\sim}{r}$. The point S is the midpoint of QR and T is the intersection of PR and OS, as shown in the diagram.



iii. Show that $\overrightarrow{OT} = \dfrac{2}{3}\underset{\sim}{r} + \dfrac{1}{3}\underset{\sim}{p}$. *(3 marks)*

iv. Using parts (ii) and (iii), or otherwise, prove that T is the point that divides the interval PR in the ratio 2 :1. *(1 mark)*

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11. Vectors, EXT2 V1 2024 HSC 14e

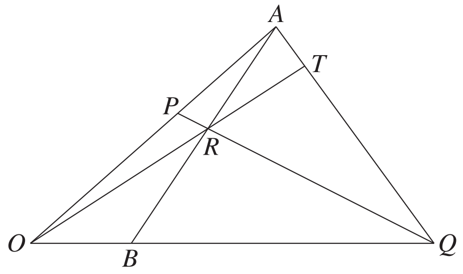
The diagram shows triangle OQA.

The point P lies on OA so that $OP : OA = 3 : 5$.

The point B lies on OQ so that $OB : OQ = 1 : 3$.

The point R is the intersection of AB and PQ.

The point T is chosen on AQ so that O, R and T are collinear.



NOT TO SCALE

Let $\underset{\sim}{a} = \overrightarrow{OA}$, $\underset{\sim}{b} = \overrightarrow{OB}$ and $\overrightarrow{PR} = k\overrightarrow{PQ}$ where k is a real number.

i. Show that $\overrightarrow{OR} = \dfrac{3}{5}(1-k)\underset{\sim}{a} + 3k\underset{\sim}{b}$. *(2 marks)*

Writing $\overrightarrow{AR} = h\overrightarrow{AB}$, where h is a real number, it can be shown that $\overrightarrow{OR} = (1-h)\underset{\sim}{a} + h\underset{\sim}{b}$. (Do NOT prove this.)

ii. Show that $k = \dfrac{1}{6}$. *(2 marks)*

iii. Find $\overrightarrow{OT}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. *(2 marks)*

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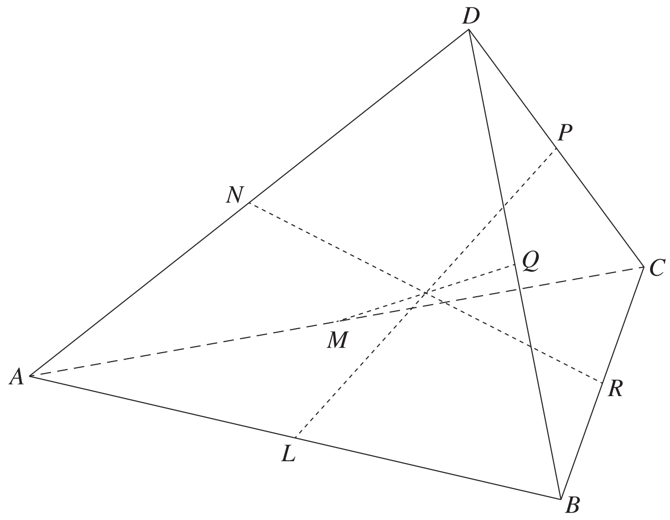
12. Vectors, EXT2 V1 2023 HSC 15b

On the triangular pyramid $ABCD$, L is the midpoint of AB, M is the midpoint of AC, N is the midpoint of AD, P is the midpoint of CD, Q is the midpoint of BD and R is the midpoint of BC.



Let $\underset{\sim}{b} = \overrightarrow{AB}$, $\underset{\sim}{c} = \overrightarrow{AC}$ and $\underset{\sim}{d} = \overrightarrow{AD}$.

i. Show that $\overrightarrow{LP} = \dfrac{1}{2}(-\underset{\sim}{b} + \underset{\sim}{c} + \underset{\sim}{d})$. *(1 mark)*

ii. It can be shown that

$$\overrightarrow{MQ} = \frac{1}{2}(\underset{\sim}{b} - \underset{\sim}{c} + \underset{\sim}{d}) \text{ and}$$

$$\overrightarrow{NR} = \frac{1}{2}(\underset{\sim}{b} + \underset{\sim}{c} - \underset{\sim}{d}). \text{ (Do NOT prove these.)}$$

Prove that

$$\left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{AC}\right|^2 + \left|\overrightarrow{AD}\right|^2 + \left|\overrightarrow{BC}\right|^2 + \left|\overrightarrow{BD}\right|^2 + \left|\overrightarrow{CD}\right|^2$$
$$= 4\left(\left|\overrightarrow{LP}\right|^2 + \left|\overrightarrow{MQ}\right|^2 + \left|\overrightarrow{NR}\right|^2\right)$$ *(3 marks)*

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13. Vectors, EXT2 V1 2021 HSC 16a

i. The point $P(x, y, z)$ lies on the sphere of radius 1 centred at the origin O.

Using the position vector of P, $\overrightarrow{OP} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$, and the triangle inequality, or otherwise, show that $|x| + |y| + |z| \geq 1$. *(2 marks)*

ii. Given the vectors $\underset{\sim}{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$, show that

$$|a_1b_1 + a_2b_2 + a_3b_3| \leq \sqrt{a_1^2 + a_2^2 + a_3^2}\ \sqrt{b_1^2 + b_2^2 + b_3^2}.$$ *(3 marks)*

iii. As in part (i), the point $P(x, y, z)$ lies on the sphere of radius 1 centred at the origin O.

Using part (ii), or otherwise, show that $|x| + |y| + |z| \leq \sqrt{3}$. *(2 marks)*

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14. Vectors, EXT2 V1 2022 HSC 14a

i. The two non-parallel vectors $\vec{u}$ and $\vec{v}$ satisfy $\lambda\vec{u} + \mu\vec{v} = \vec{0}$ for some real numbers λ and μ.

Show that $\lambda = \mu = 0$. *(2 marks)*

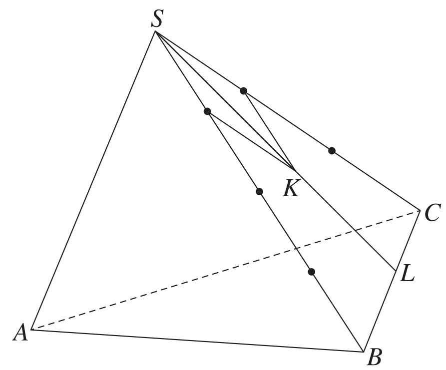
ii. The two non-parallel vectors $\vec{u}$ and $\vec{v}$ satisfy $\lambda_1\vec{u} + \mu_1\vec{v} = \lambda_2\vec{u} + \mu_2\vec{v}$ for some real numbers $\lambda_1, \lambda_2, \mu_1$ and μ_2.

Using part (i), or otherwise, show that $\lambda_1 = \lambda_2$ and $\mu_1 = \mu_2$. *(1 mark)*

The diagram below shows the tetrahedron with vertices A, B, C and S.

The point K is defined by $\overrightarrow{SK} = \frac{1}{4}\overrightarrow{SB} + \frac{1}{3}\overrightarrow{SC}$, as shown in the diagram.

The point L is the point of intersection of the straight lines SK and BC.



iii. Using part (ii), or otherwise, determine the position of L by showing that $\overrightarrow{BL} = \frac{4}{7}\overrightarrow{BC}$. *(2 marks)*

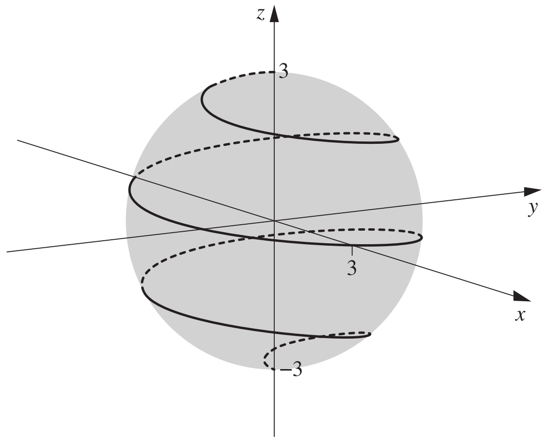
iv. The point P is defined by $\overrightarrow{AP} = -6\overrightarrow{AB} - 8\overrightarrow{AC}$.

Does P lie on the line AL? Justify your answer. *(2 marks)*

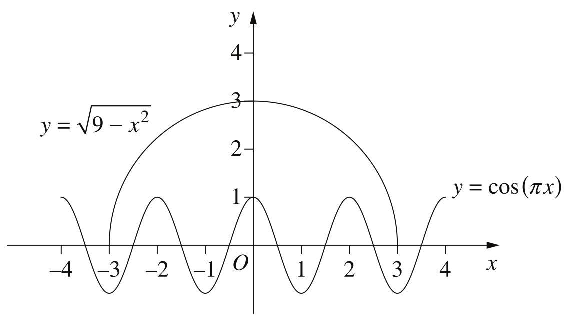
15. Vectors, EXT2 V1 2023 HSC 15c

A curve $\mathcal{C}$ spirals 3 times around the sphere centred at the origin and with radius 3, as shown.

A particle is initially at the point $(0, 0, -3)$ and moves along the curve $\mathcal{C}$ on the surface of the sphere, ending at the point $(0, 0, 3)$.



By using the diagram below, which shows the graphs of the functions $f(x) = \cos(\pi x)$ and $g(x) = \sqrt{9 - x^2}$, and considering the graph $y = f(x)g(x)$, give a possible set of parametric equations that describe the curve $\mathcal{C}$. ***(3 marks)***



Worked Solutions

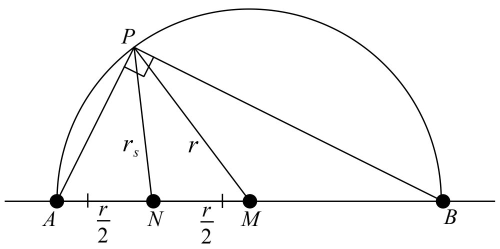
1. Vectors, EXT2 V1 2021 HSC 1 MC

$$\overrightarrow{AB} + \overrightarrow{CQ} = \overrightarrow{CD} + \overrightarrow{DP}$$
$$= \overrightarrow{CP}$$

$\Rightarrow\ B$

2. Vectors, EXT2 V1 2022 HSC 9 MC

Diagram below is a 2-D sliced image of the 3-D geometry:



$\overrightarrow{AP} \cdot \overrightarrow{BP} = 0.$

$r = \dfrac{\left|\overrightarrow{AB}\right|}{2}$

S_2 includes N where $\left|\overrightarrow{AN}\right| = \left|\overrightarrow{MN}\right| = \dfrac{r}{2}$

Let $r_s =$ radius of S

Point P is intersection of S_1 and S_2

♦♦♦ Mean mark 20%.

By Pythagoras (see diagram):

$$r_s^2 = r^2 - (\frac{r}{2})^2$$

$$= \frac{3r^2}{4}$$

$$r_s = \frac{\sqrt{3}}{2} \times r$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\left|\overrightarrow{AB}\right|}{2}$$

$$= \frac{\sqrt{3}\left|\overrightarrow{AB}\right|}{4}$$

$=> D$

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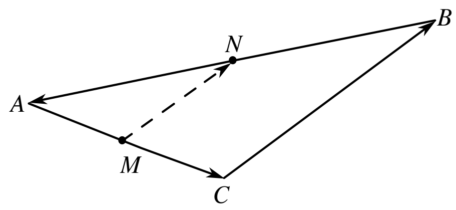
3. Vectors, EXT2 V1 EQ-Bank 14

a.



$\overrightarrow{AC} + \overrightarrow{CB} + \overrightarrow{BA} = 0$ (resultant vector ends at starting point)

$\overrightarrow{CB} = -\left(\overrightarrow{AC} + \overrightarrow{BA}\right)$

Since M and N are midpoints:

$$\overrightarrow{MN} = -\frac{1}{2}\overrightarrow{AC} - \frac{1}{2}\overrightarrow{BA}$$

$$= -\frac{1}{2}\left(\overrightarrow{AC} + \overrightarrow{AB}\right)$$

$$= \frac{1}{2}\overrightarrow{CB} \ \ldots \text{ as required}$$

b. If $\vec{u} = k\vec{v}$ (k scalar) $\Rightarrow$ $\vec{u} \,||\, \vec{v}$

$\overrightarrow{MN} = \frac{1}{2}\overrightarrow{CB}$ (see (i))

$\therefore MN \,||\, RCB$

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4. Vectors, EXT2 V1 EQ-Bank 19

a. $A(3,0,0), \quad G(0,3,3)$

$$\overrightarrow{AG} = \begin{pmatrix} 0 \\ 3 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix}$$

$$\left| \overrightarrow{AG} \right| = \sqrt{9+9+9} = 3\sqrt{3} \text{ units}$$

b. $H(3,3,3)$

$$\overrightarrow{BH} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$$

$$\overrightarrow{AG} \cdot \overrightarrow{BH} = \left| \overrightarrow{AG} \right| \cdot \left| \overrightarrow{BH} \right| \cos\theta$$

$$\begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} = \sqrt{9+9+9} \cdot \sqrt{9+9+9}\cos\theta$$

$$-9+9+9 = 27\cos\theta$$

$$\cos\theta = \frac{1}{3}$$

$$\theta = 70.52\ldots = 70^\circ 32'$$

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5. Vectors, EXT2 V1 2025 HSC 15a

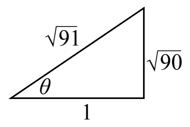
$$A = 2 \times \left[\frac{1}{2} |a| |b| \sin\theta \right]$$

$$\underset{\sim}{a} = 4\underset{\sim}{i} + 3\underset{\sim}{j} - \underset{\sim}{k} \ \Rightarrow \ \left|\underset{\sim}{a}\right| = \sqrt{16+9+1} = \sqrt{26}$$

$$\underset{\sim}{b} = 2\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k} \ \Rightarrow \ \left|\underset{\sim}{b}\right| = \sqrt{4+1+9} = \sqrt{14}$$

$$\underset{\sim}{a} \cdot \underset{\sim}{b} = \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = 8 - 3 - 3 = 2$$

$$\cos\theta = \frac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{\left|\underset{\sim}{a}\right| \left|\underset{\sim}{b}\right|} = \frac{2}{\sqrt{26}\sqrt{14}} = \frac{1}{\sqrt{91}}$$



$$\sin\theta = \frac{\sqrt{90}}{\sqrt{91}}$$

$$\therefore A = 2 \times \frac{1}{2} \times \sqrt{26} \times \sqrt{14} \times \frac{\sqrt{90}}{\sqrt{91}} = 6\sqrt{10} \text{ units}^2$$

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6. Vectors, EXT2 V1 EQ-Bank 22

Scalar Product result 1:

$$\underset{\sim}{a}\cdot\underset{\sim}{b} = x_1x_2 + y_1y_2 + z_1z_2$$

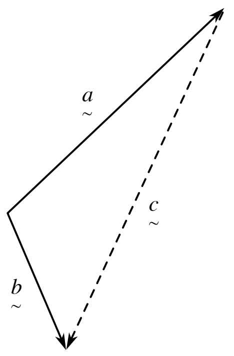
$$= 3\times 4 + (-2)\times 3 + 1\times(-4)$$

$$= 2$$

Scalar Product result 2:

$$\underset{\sim}{a}\cdot\underset{\sim}{b} = \left|\underset{\sim}{a}\right|\left|\underset{\sim}{b}\right|\cos\theta$$

Let $\underset{\sim}{c} = \underset{\sim}{b} - \underset{\sim}{a}$



$$\underset{\sim}{c} = \begin{pmatrix} 4 \\ 3 \\ -4 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ -5 \end{pmatrix}$$

$$\left|\underset{\sim}{c}\right| = \sqrt{1^2+5^2+(-5)^2} = \sqrt{51}$$

$$\left|\underset{\sim}{a}\right| = \sqrt{3^2+(-2)^2+1^2} = \sqrt{14}$$

$$\left|\underset{\sim}{b}\right| = \sqrt{4^2+3^2+(-4)^2} = \sqrt{41}$$

Using cosine rule: $c^2 = a^2 + b^2 - 2ab\,\cos C$

$$\Rightarrow\quad ab\,\cos C = \frac{a^2+b^2-c^2}{2}$$

$$\underset{\sim}{a}\cdot\underset{\sim}{b} = \left|\underset{\sim}{a}\right|\left|\underset{\sim}{b}\right|\cos\theta$$

$$= \frac{\left|\underset{\sim}{a}\right|^2 + \left|\underset{\sim}{b}\right|^2 - \left|\underset{\sim}{c}\right|^2}{2}$$

$$= \frac{14+41-51}{2}$$

$$= 2$$

$$\therefore\ \underset{\sim}{a}\cdot\underset{\sim}{b} = \left|\underset{\sim}{a}\right|\left|\underset{\sim}{b}\right|\cos\theta = x_1x_2 + y_1y_2 + z_1z_2$$

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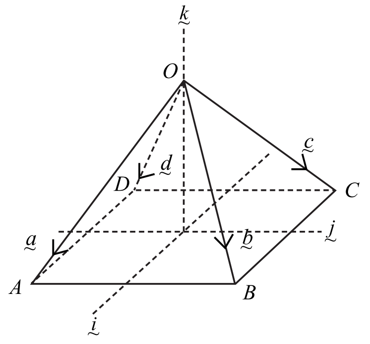
7. Vectors, EXT2 V1 EQ-Bank 25



Let $A = (p, -p, -k)$,

$$\underset{\sim}{a} = \overrightarrow{OA} = p\underset{\sim}{i} - p\underset{\sim}{j} - q\underset{\sim}{k}$$

$$\underset{\sim}{b} = \overrightarrow{OB} = p\underset{\sim}{i} + p\underset{\sim}{j} - q\underset{\sim}{k}$$

$$\underset{\sim}{c} = \overrightarrow{OC} = -p\underset{\sim}{i} + p\underset{\sim}{j} - q\underset{\sim}{k}$$

$$\underset{\sim}{d} = \overrightarrow{OA} = -p\underset{\sim}{i} - p\underset{\sim}{j} - q\underset{\sim}{k}$$

$$\underset{\sim}{a} + \underset{\sim}{c} = -2q\underset{\sim}{k}$$

$$\underset{\sim}{b} + \underset{\sim}{d} = -2q\underset{\sim}{k}$$

$$\therefore\ \underset{\sim}{a} + \underset{\sim}{c} = \underset{\sim}{b} + \underset{\sim}{d}$$

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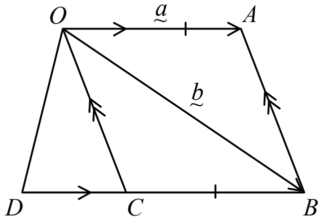
8. Vectors, EXT2 V1 EQ-Bank 27



$$\overrightarrow{DA} = \overrightarrow{DB} + \overrightarrow{BA}$$

$$\overrightarrow{BA} = \underset{\sim}{a} - \underset{\sim}{b}$$

Since $OC \,||\, AB$ and $OA \,||\, CB$

$\Rightarrow OABC$ is a parallelogram

$$\overrightarrow{OA} = \overrightarrow{CB} = \underset{\sim}{a}$$

$$\overrightarrow{DC} = \frac{1}{2}\underset{\sim}{a}$$

$$\begin{aligned}\overrightarrow{DB} &= \overrightarrow{DC} + \overrightarrow{CB}\\ &= \frac{3}{2}\underset{\sim}{a}\end{aligned}$$

$$\begin{aligned}\therefore\ \overrightarrow{DA} &= \frac{3}{2}\underset{\sim}{a} + \left(\underset{\sim}{a} - \underset{\sim}{b}\right)\\ &= \frac{5}{2}\underset{\sim}{a} - \underset{\sim}{b}\end{aligned}$$

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9. Vectors, EXT2 V1 2021 HSC 12e

i. Since diagonal $\overrightarrow{AC}$ is bisected by H:

$$\overrightarrow{HA} = -\ \overrightarrow{HC}$$

Similarly for diagonal $\overrightarrow{BD}$

$$\overrightarrow{HB} = -\overrightarrow{HD}$$

$$\begin{aligned}\therefore\ \overrightarrow{HA} + \overrightarrow{HB} + \overrightarrow{HC} + \overrightarrow{HD} &= -\overrightarrow{HC} - \overrightarrow{HD} + \overrightarrow{HC} + \overrightarrow{HD}\\ &= \underset{\sim}{0}\end{aligned}$$

ii. $\overrightarrow{GA} = \overrightarrow{GH} + \overrightarrow{HA}$, $\overrightarrow{GB} = \overrightarrow{GH} + \overrightarrow{HB}$
$\overrightarrow{GC} = \overrightarrow{GH} + \overrightarrow{HC}$, $\overrightarrow{GD} = \overrightarrow{GH} + \overrightarrow{HD}$

$$\begin{aligned}&\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} + \overrightarrow{GD} + \overrightarrow{GS}\\ &\quad = \overrightarrow{GH} + \overrightarrow{HA} + \overrightarrow{GH} + \overrightarrow{HB} + \overrightarrow{GH} + \overrightarrow{HC} + \overrightarrow{GH} + \overrightarrow{HD} + \overrightarrow{GS}\\ &\quad = 4\overrightarrow{GH} + \left(\overrightarrow{HA} + \overrightarrow{HB} + \overrightarrow{HC} + \overrightarrow{HD} + \overrightarrow{GS}\right)\\ &\quad = 4\overrightarrow{GH} + \overrightarrow{GS}\end{aligned}$$

$$\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} + \overrightarrow{GD} + \overrightarrow{GS} = \underset{\sim}{0} \quad \text{(given)}$$

$$\therefore 4\overrightarrow{GH} + \overrightarrow{GS} = \underset{\sim}{0}$$

iii. $\overrightarrow{HS} = \overrightarrow{HG} + \overrightarrow{GS}$
$\overrightarrow{GS} = \overrightarrow{HS} + \overrightarrow{GH}$

♦ *Mean mark 48%.*

Using part (ii):

$$4\overrightarrow{GH} + \overrightarrow{HS} + \overrightarrow{GH} = \underset{\sim}{0}$$

$$5\overrightarrow{GH} = \overrightarrow{SH}$$

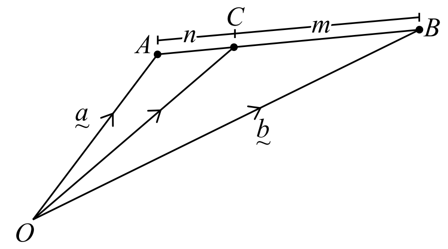
$$\overrightarrow{GH} = \frac{1}{5}\overrightarrow{SH}$$

$$\overrightarrow{HG} = \frac{1}{5}\overrightarrow{HS}$$

$$\therefore\ \lambda = \frac{1}{5}$$

10. Vectors, EXT2 V1 2020 HSC 15b

i.



$$\frac{\overrightarrow{AC}}{\overrightarrow{AB}} = \frac{n}{m+n}$$

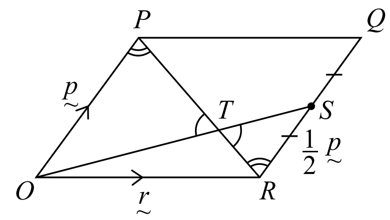
$$\begin{aligned}\overrightarrow{AC} &= \frac{n}{m+n} \cdot \overrightarrow{AB}\\ &= \frac{n}{m+n}\left(\underset{\sim}{b} - \underset{\sim}{a}\right)\end{aligned}$$

ii. $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$

$$\begin{aligned}&= \underset{\sim}{a} + \frac{n}{m+n}\left(\underset{\sim}{b} - \underset{\sim}{a}\right)\\ &= \underset{\sim}{a} - \frac{n}{m+n}\underset{\sim}{a} + \frac{n}{m+n}\underset{\sim}{b}\\ &= \left(1 - \frac{n}{m+n}\right)\underset{\sim}{a} + \frac{n}{m+n}\underset{\sim}{b}\\ &= \left(\frac{m+n-n}{m+n}\right)\underset{\sim}{a} + \frac{n}{m+n}\underset{\sim}{b}\\ &= \frac{m}{m+n}\underset{\sim}{a} + \frac{n}{m+n}\underset{\sim}{b}\end{aligned}$$

iii. Show $\overrightarrow{OT} = \frac{2}{3}\underset{\sim}{r} + \frac{1}{3}\underset{\sim}{p}$

♦ *Mean mark part (iii) 50%.*



Consider ΔPTO and ΔRTS:

$\angle PTO = \angle RTS$ (vertically opposite)

$\angle OPT = \angle SRT$ (vertically opposite)

$\therefore\ \Delta PTO \mid\mid\mid \Delta RTS$ (equiangular)

$OT : TS = OP : SR = 2 : 1$

(corresponding sides in the same ratio)

$$\frac{\overrightarrow{OT}}{\overrightarrow{OS}} = \frac{2}{3}$$

$$\begin{aligned}\overrightarrow{OT} &= \frac{2}{3}\overrightarrow{OS}\\ &= \frac{2}{3}\left(\underset{\sim}{r} + \frac{1}{2}\underset{\sim}{p}\right)\\ &= \frac{2}{3}\underset{\sim}{r} + \frac{1}{3}\underset{\sim}{p}\end{aligned}$$

Mean mark part (iv) 51%.

iv. Let $\overrightarrow{OT}$ divide PR so that $\dfrac{TR}{PT} = \dfrac{m}{n}$

Using part (ii):

$$\overrightarrow{OT} = \frac{m}{m+n}\underset{\sim}{p} + \frac{n}{m+n}c$$

$$\overrightarrow{OT} = \frac{1}{3}\underset{\sim}{p} + \frac{2}{3}\underset{\sim}{r} \quad \text{(part (iii))}$$

$$\frac{m}{m+n} = \frac{1}{3},\ \frac{n}{m+n} = \frac{2}{3}$$

$\Rightarrow\ m = 1\ ,\ n = 2$

$\therefore\ T$ divides PR in ratio $2:1$.

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11. Vectors, EXT2 V1 2024 HSC 14e

i. $\underset{\sim}{a} = \overrightarrow{OA},\ \underset{\sim}{b} = \overrightarrow{OB},\ \overrightarrow{PR} = k\overrightarrow{PQ}$

$\overrightarrow{OP} : \overrightarrow{OA} = 3:5\ \Rightarrow\ \overrightarrow{OP} = \dfrac{3}{5} \times \overrightarrow{OA}$

$\overrightarrow{OB} : \overrightarrow{OQ} = 1:3\ \Rightarrow\ \overrightarrow{OQ} = 3 \times \overrightarrow{OB}$

Show $\overrightarrow{OR} = \dfrac{3}{5}(1-k)\underset{\sim}{a} + 3k\underset{\sim}{b}$

$$\begin{aligned}\overrightarrow{OR} &= \overrightarrow{OP} + k\overrightarrow{PQ}\\ &= \overrightarrow{OP} + k(\overrightarrow{OQ} - \overrightarrow{OP})\\ &= (1-k)\overrightarrow{OP} + k\overrightarrow{OQ}\\ &= \frac{3}{5}(1-k)\underset{\sim}{a} + 3k\underset{\sim}{b}\end{aligned}$$

ii. $\overrightarrow{AR} = h\overrightarrow{AB},\quad h \in \mathbb{R}$

$\overrightarrow{OR} = (1-h)\underset{\sim}{a} + h\underset{\sim}{b}$ (given)

$\overrightarrow{OR} = \dfrac{3}{5}(1-k)\underset{\sim}{a} + 3k\underset{\sim}{b}$ (part(i))

Vector $\underset{\sim}{a} \neq \lambda\underset{\sim}{b}\ (\lambda \in \mathbb{R})\ \Rightarrow$ linearly independent and are basis vectors for $\overrightarrow{OR}$.

Equating coefficients:

$\dfrac{3}{5}(1-k) = 1 - h \ldots (1)$

$h = 3k \ldots (2)$

Substituting (2) into (1)

$$\begin{aligned}\frac{3}{5}(1-k) &= 1 - 3k\\ 3 - 3k &= 5 - 15k\\ k &= \frac{1}{6}\end{aligned}$$

♦ Mean mark (iii) 45%.

iii. $\overrightarrow{OT} = \lambda\overrightarrow{OR}$

Using parts (i) and (ii):

$$\overrightarrow{OR} = \frac{3}{5}\left(1 - \frac{1}{6}\right)\underset{\sim}{a} + 3\left(\frac{1}{6}\right)\underset{\sim}{b} = \frac{1}{2}\left(\underset{\sim}{a} + \underset{\sim}{b}\right)$$

$$\overrightarrow{OT} = \frac{\lambda}{2}(\underset{\sim}{a} + \underset{\sim}{b})$$

Find λ:

$$\begin{aligned}\overrightarrow{OT} &= \overrightarrow{OA} + \mu\overrightarrow{AQ} \\ \frac{\lambda}{2}(\underset{\sim}{a} + \underset{\sim}{b}) &= \underset{\sim}{a} + \mu(3\underset{\sim}{b} - \underset{\sim}{a}) \quad \left(\text{noting } \overrightarrow{AQ} = \overrightarrow{OQ} - \overrightarrow{OA} = 3\underset{\sim}{b} - \underset{\sim}{a}\right) \\ &= \underset{\sim}{a}(1 - \mu) + 3\mu\underset{\sim}{b}\end{aligned}$$

Equating coefficients:

$$\frac{\lambda}{2} = 3\mu \;\Rightarrow\; \mu = \frac{\lambda}{6}$$

$$\begin{aligned}1 - \frac{\lambda}{6} &= \frac{\lambda}{2} \\ 6 - \lambda &= 3\lambda \\ \lambda &= \frac{3}{2}\end{aligned}$$

$$\therefore \overrightarrow{OT} = \frac{3}{4}(\underset{\sim}{a} + \underset{\sim}{b})$$

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12. **Vectors, EXT2 V1 2023 HSC 15b**

i. $\overrightarrow{LP} = \overrightarrow{LA} + \overrightarrow{AC} + \overrightarrow{CP}$

$$= \frac{1}{2}\overrightarrow{BA} + \overrightarrow{AC} + \frac{1}{2}\overrightarrow{CD}$$

$$= -\frac{1}{2}\underset{\sim}{b} + \underset{\sim}{c} + \frac{1}{2}(\underset{\sim}{d} - \underset{\sim}{c})$$

$$= -\frac{1}{2}\underset{\sim}{b} + \underset{\sim}{c} + \frac{1}{2}\underset{\sim}{d} - \frac{1}{2}\underset{\sim}{c}$$

$$= \frac{1}{2}(-\underset{\sim}{b} + \underset{\sim}{c} + \underset{\sim}{d})$$

ii. $\overrightarrow{MQ} = \frac{1}{2}(\underset{\sim}{b} - \underset{\sim}{c} + \underset{\sim}{d})$

♦♦ *Mean mark (ii) 34%.*

$\overrightarrow{NR} = \frac{1}{2}(\underset{\sim}{b} + \underset{\sim}{c} - \underset{\sim}{d}).$

$$\text{RHS} = 4\left(\left|\overrightarrow{LP}\right|^2 + \left|\overrightarrow{MQ}\right|^2 + \left|\overrightarrow{NR}\right|^2\right)$$

$$= 4\left(\overrightarrow{LP}\cdot\overrightarrow{LP} + \overrightarrow{MQ}\cdot\overrightarrow{MQ} + \overrightarrow{NR}\cdot\overrightarrow{NR}\right)$$

$$= 4\Bigg(\frac{1}{4}(-\underset{\sim}{b} + \underset{\sim}{c} + \underset{\sim}{d})\cdot(-\underset{\sim}{b} + \underset{\sim}{c} + \underset{\sim}{d}) +$$

$$\frac{1}{4}(\underset{\sim}{b} - \underset{\sim}{c} + \underset{\sim}{d})\cdot(\underset{\sim}{b} - \underset{\sim}{c} + \underset{\sim}{d}) +$$

$$\frac{1}{4}(\underset{\sim}{b} + \underset{\sim}{c} - \underset{\sim}{d})\cdot(\underset{\sim}{b} + \underset{\sim}{c} - \underset{\sim}{d})\Bigg)$$

$$= (\underset{\sim}{c} + (\underset{\sim}{d} - \underset{\sim}{b}))\cdot(\underset{\sim}{c} + (\underset{\sim}{d} - \underset{\sim}{b})) + (\underset{\sim}{d} + (\underset{\sim}{b} - \underset{\sim}{c}))\cdot(\underset{\sim}{d} + (\underset{\sim}{b} - \underset{\sim}{c})) +$$

$$(\underset{\sim}{b} + (\underset{\sim}{c} - \underset{\sim}{d}))\cdot(\underset{\sim}{b} + (\underset{\sim}{c} - \underset{\sim}{d}))$$

$$= |\underset{\sim}{c}|^2 + 2\underset{\sim}{c}(\underset{\sim}{d} - \underset{\sim}{b}) + |\underset{\sim}{d} - \underset{\sim}{b}|^2 +$$

$$|\underset{\sim}{d}|^2 + 2\underset{\sim}{d}(\underset{\sim}{b} - \underset{\sim}{c}) + |\underset{\sim}{b} - \underset{\sim}{c}|^2 +$$

$$|\underset{\sim}{b}|^2 + 2\underset{\sim}{b}(\underset{\sim}{c} - \underset{\sim}{d}) + |\underset{\sim}{c} - \underset{\sim}{d}|^2$$

$$= |\underset{\sim}{b}|^2 + |\underset{\sim}{c}|^2 + |\underset{\sim}{d}|^2 + |\underset{\sim}{b} - \underset{\sim}{c}|^2 + |\underset{\sim}{d} - \underset{\sim}{b}|^2 + |\underset{\sim}{c} - \underset{\sim}{d}|^2 +$$

$$2(\underset{\sim}{c}\cdot\underset{\sim}{d} - \underset{\sim}{c}\cdot\underset{\sim}{b} + \underset{\sim}{d}\cdot\underset{\sim}{b} - \underset{\sim}{d}\cdot\underset{\sim}{c} + \underset{\sim}{b}\cdot\underset{\sim}{c} - \underset{\sim}{b}\cdot\underset{\sim}{d})$$

$$= \left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{AC}\right|^2 + \left|\overrightarrow{AD}\right|^2 + \left|\overrightarrow{BC}\right|^2 + \left|\overrightarrow{BD}\right|^2 + \left|\overrightarrow{CD}\right|^2 + 0$$

$$= \text{LHS}$$

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13. Vectors, EXT2 V1 2021 HSC 16a

i. Triangle inequality: $|x| + |y| \geq |x + y|$

♦ *Mean mark (i) 50%.*

$$|x| + |y| + |z| = \left|x\underset{\sim}{i}\right| + \left|y\underset{\sim}{j}\right| + \left|z\underset{\sim}{k}\right|$$

$$\geq \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| + \left|z\underset{\sim}{k}\right|$$

$$\geq \left|x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}\right|$$

$$\geq 1 \quad \left(\left|\overrightarrow{OP}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}\right| = 1\right)$$

ii. Using the dot product:

♦ *Mean mark (ii) 42%.*

$$\underset{\sim}{a}\cdot\underset{\sim}{b} = a_1b_1 + a_2b_2 + a_3b_3$$

$$\underset{\sim}{a}\cdot\underset{\sim}{b} = \left|\underset{\sim}{a}\right|\left|\underset{\sim}{b}\right|\cos\theta$$

$$a_1b_1 + a_2b_2 + a_3b_3 = \sqrt{a_1^2 + a_2^2 + a_3^2}\cdot\sqrt{b_1^2 + b_2^2 + b_3^2}\cdot\cos\theta$$

$$|a_1b_1 + a_2b_2 + a_3b_3| = \sqrt{a_1^2 + a_2^2 + a_3^2}\cdot\sqrt{b_1^2 + b_2^2 + b_3^2}\cdot|\cos\theta|$$

Since $-1 \leq \cos\theta \leq 1 \ \Rightarrow\ |\cos\theta| \leq 1$

$$\therefore\ |a_1b_1 + a_2b_2 + a_3b_3| \leq \sqrt{a_1^2 + a_2^3 + a_3^2}\cdot\sqrt{b_1^2 + b_2^2 + b_3^2}$$

iii. Using part (ii) with vectors:

♦♦♦ *Mean mark (iii) 14%.*

$$\underset{\sim}{a} = \begin{pmatrix}1\\1\\1\end{pmatrix},\ \underset{\sim}{b} = \begin{pmatrix}|x|\\|y|\\|z|\end{pmatrix}$$

$$\big|\,|x| + |y| + |z|\,\big| \leq \sqrt{1^2 + 1^2 + 1^2}\cdot\sqrt{x^2 + y^2 + z^2}$$

$$|x| + |y| + |z| \leq \sqrt{3}$$

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14. Vectors, EXT2 V1 2022 HSC 14a

i. $\lambda\vec{u} + \mu\vec{v} = 0$

$$\lambda\vec{u} = -\mu\vec{v}$$

$$\lambda = 0 \quad \text{or} \quad \vec{u} = -\left(\frac{\mu}{\lambda}\right)\vec{v} = k\vec{v} \quad (k \in \mathbb{R})$$

Since $\vec{u}$ and $\vec{v}$ are not parallel

$$\Rightarrow \lambda = \mu = 0$$

ii. $\lambda_1\vec{u} + \mu_1\vec{v} = \lambda_2\vec{u} + \mu_2\vec{v}$

$$\lambda_1\vec{u} - \lambda_2\vec{u} + \mu_1\vec{v} - \mu_2\vec{v} = \vec{0}$$

$$(\lambda_1 - \lambda_2)\vec{u} + (\mu_1 - \mu_2)\vec{v} = \vec{0}$$

Using part (i):

$$(\lambda_1 - \lambda_2) = 0 \text{ and } (\mu_1 - \mu_2) = 0$$

$$\therefore \lambda_1 = \lambda_2 \quad \text{and} \quad \mu_1 = \mu_2 \ \ldots \text{as required}$$

Mean mark (ii) 56%.

iii. $\overrightarrow{BL} = \lambda\overrightarrow{BC}$

$$= \lambda\left(\overrightarrow{BS} + \overrightarrow{SC}\right) \quad \ldots (1)$$

$$\overrightarrow{BL} = \overrightarrow{BS} + \mu\overrightarrow{SK}$$

$$= -\overrightarrow{SB} + \mu\left(\frac{1}{4}\overrightarrow{SB} + \frac{1}{3}\overrightarrow{SC}\right)$$

$$= -\overrightarrow{SB} + \frac{\mu}{4}\overrightarrow{SB} + \frac{\mu}{3}\overrightarrow{SC}$$

$$= \frac{\mu}{3}\overrightarrow{SC} + \left(\frac{\mu}{4} - 1\right)\overrightarrow{SB} \quad \ldots (2)$$

♦♦♦ Mean mark (iii) 25%.

Using $(1) = (2)$:

$$\lambda\overrightarrow{BS} + \lambda\overrightarrow{SC} = \frac{\mu}{3}\overrightarrow{SC} + \left(\frac{\mu}{4} - 1\right)\overrightarrow{SB}$$

$$\frac{\mu}{3} = \lambda, \quad 1 - \frac{\mu}{4} = \lambda$$

$$\frac{\mu}{3} = 1 - \frac{\mu}{4}$$

$$\frac{4\mu + 3\mu}{12} = 1$$

$$\frac{7\mu}{12} = 1$$

$$\mu = \frac{12}{7}$$

$$\lambda = \frac{\frac{12}{7}}{3} = \frac{4}{7}$$

$$\therefore \overrightarrow{BL} = \frac{4}{7}\overrightarrow{BC} \quad \ldots \text{as required}$$

iv. $\overrightarrow{AP} = -6\overrightarrow{AB} - 8\overrightarrow{AC}$

♦♦♦ Mean mark (iv) 23%.

If P lies on $AL, \exists k$ such that $\overrightarrow{AP} = k\overrightarrow{AL}$

$$-6\overrightarrow{AB} - 8\overrightarrow{AC} = k\overrightarrow{AL}$$

$$-6\overrightarrow{AB} - 8\left(\overrightarrow{AB} + \overrightarrow{BC}\right) = k\left(\overrightarrow{AB} + \overrightarrow{BL}\right)$$

$$-14\overrightarrow{AB} - 8\overrightarrow{BC} = k\left(\overrightarrow{AB} + \frac{4}{7}\overrightarrow{BC}\right) \quad \text{(see part (iii))}$$

$$-14\overrightarrow{AB} - 8\overrightarrow{BC} = k\overrightarrow{AB} + \frac{4k}{7}\overrightarrow{BC}$$

$$k = -14, \quad \frac{4k}{7} = -8 \quad \Rightarrow \quad k = -14$$

$$\therefore \ P \text{ lies on } \ AL.$$

15. Vectors, EXT2 V1 2023 HSC 15c

Since the curve lies on a sphere with radius 3:

♦♦♦ Mean mark 22%.

$x^2+y^2+z^2=3^3$

Considering the graph $y=\cos(\pi t)\sqrt{9-t^2}$ (as per hint)

$\left(\cos(\pi t)\sqrt{9-t^2}\right)^2+\left(\sin(\pi t)\sqrt{9-t^2}\right)^2+t^2=3^2\ \ldots(1)$

Since z increases and x and y change signs

$\Rightarrow z=t$

In order to satisfy the equation in (1):

x, y must be one of $\pm\cos(\pi t)\sqrt{9-t^2}$ or $\pm\sin(\pi t)\sqrt{9-t^2}$

At $z=0,\ t=0,\ x=3$ (from graph):

$\Rightarrow x=\cos(\pi t)\sqrt{9-t^2}$

At $z=0+\epsilon,\ t=0+\epsilon,\ y<0$ (from graph):

$\Rightarrow y=-\sin(\pi t)\sqrt{9-t^2}$