MATHEMATICS EXTENSION 1

SmarterEd Maths & Sciences

HSC REVISION SERIES 2026

Welcome to the SmarterMaths 2026 HSC Extension 1 Revision Series.

Key features of this year's Revision Series include:

- ~18 hours of carefully selected HSC revision questions
- Targeted practice across the most important HSC question types
- Designed for students aiming for a Band 5-6 result
- Higher weighting towards historically difficult question types
- Topic and question weightings reflect historical HSC exam allocations
- Build your own personalised revision resource by ***carefully reviewing and annotating*** these worksheets throughout Term 3.

**Philosophy: Every question has earned its place. Historical HSC trends, common question types and past areas of student difficulty have all influenced the selection process.*

IMPORTANT

If you've already completed some of these questions during the year, **that's a good thing.**

Many of the highest-performing students credit repeated practice with challenging past HSC questions as a key ingredient in their HSC success. This revision series focuses on the highest-value question types, helping you build confidence and critically, *speed* through the exam.

HSC Final Study – Extension 1: Calculus

Worksheet 4 of 5 | Estimated exam weighting: 35.0%

Syllabus: C3 Applications of Calculus 1

Key Areas addressed by this worksheet:

Further Area and Solids of Revolution (6.4%)

- *Further Area and Solids of Revolution* has been a sneaky large contributor to new syllabus Ext1 exams, appearing in the longer answer section of every new syllabus exam to date, including two separate questions in four of those years!
- *Solids of Revolution* **(★)** are the dominant question type. *x*-axis and *y*-axis rotations are both reviewed, with the latter historically causing more problems.
- This area is typically examined at the band 4 difficulty although this was ratcheted up by *2023 Q12e* **(★)** which is a "must review" question.
- Question choice includes recent *Ext1* examples and others from the 2013-14 *Ext1* papers. The rest of the questions are chosen from past Advanced papers (identified by *EXT1** in their title).
- *Further Area* **(★)** has appeared in three exams and demands its own revision attention.

KEY: ★ Priority Revision – content or specific questions flagged by SM analysis as key revision.

"The SmarterMaths HSC exam preparation courses are incredible resources."

~ Peter Hargraves
James Sheahan Catholic High School

EXTENSION 1 2026

HSC Revision Series

Calculus (4 of 5)
C3 Applications of Calculus 1
Further Area and Solids of Revolution

Exam Equivalent Time: 75 minutes *(based on allocation of 1.5 minutes per mark)*

Questions

1. Calculus, EXT1 C3 2025 HSC 6 MC

Given that a is a non-zero constant, which of the following integrals is equal to zero?

A. $\displaystyle\int_{-a}^{a} x\cos^{-1}(x)\,dx$

B. $\displaystyle\int_{-a}^{a} x^2\cos^{-1}(x)\,dx$

C. $\displaystyle\int_{-a}^{a} x\tan^{-1}(x)\,dx$

D. $\displaystyle\int_{-a}^{a} x^2\tan^{-1}(x)\,dx$

2. Calculus, EXT1 C3 2025 HSC 12b

Consider the region bounded by the hyperbola $y=\dfrac{1}{x}$, the y-axis and the lines $y=1$ and $y=a$ for $a>1$.

Find the volume of the solid of revolution formed when the region is rotated about the y-axis. *(2 marks)*

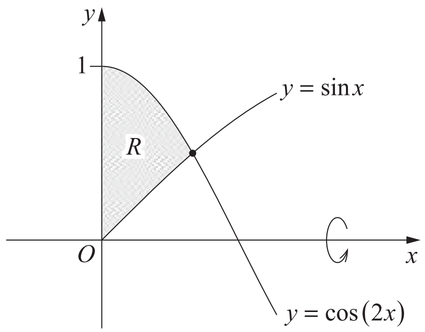
3. Calculus, EXT1 C3 EQ-Bank 15

a. Sketch the region bounded by the curve $y=x^2$ and the lines $y=16$ and $y=9$. *(1 mark)*

b. Calculate the area of this region. *(3 marks)*

4. Calculus, EXT1 C3 2020 HSC 13b

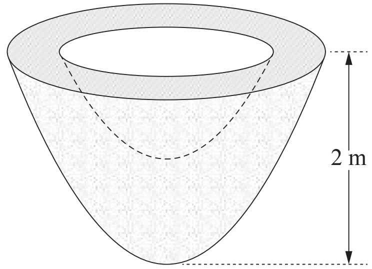
The region R is bounded by the y-axis, the graph of $y=\cos(2x)$ and the graph of $y=\sin x$, as shown in the diagram.



Find the volume of the solid of revolution formed when the region R is rotated about the x-axis. *(4 marks)*

5. Calculus, EXT1 C3 2021 HSC 13a

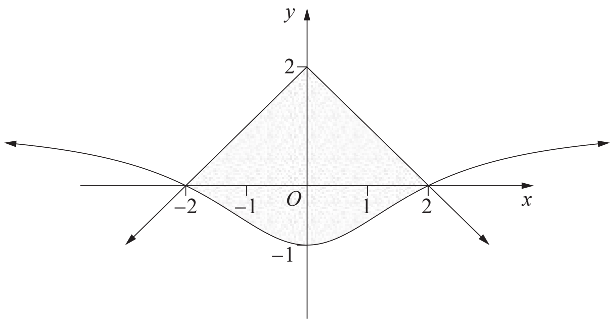
A 2-metre-high sculpture is to be made out of concrete. The sculpture is formed by rotating the region between $y=x^2$, $y=x^2+1$ and $y=2$ around the y-axis.



Find the volume of concrete needed to make the sculpture. *(3 marks)*

6. Calculus, EXT1 C3 2021 HSC 13c

The region enclosed by $y = 2 - |x|$ and $y = 1 - \dfrac{8}{4 + x^2}$ is shaded in the diagram.



Find the exact value of the area of the shaded region. *(3 marks)*

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7. Calculus, EXT1 C3 EQ-Bank 21

The parabola with equation $y = 9 - x^2$ cuts the y-axis at $P(0, 9)$ and the x-axis at $Q(3, 0)$.

Find the exact volume of the solid of revolution formed when the area between the line PQ and the parabola is rotated about the y-axis. *(4 marks)*

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8. Calculus, EXT1 C3 EQ-Bank 22

Find the volume of the solid of revolution formed when the graph of $y = \sqrt{\dfrac{1 + 2x}{1 + x^2}}$ is rotated about the x-axis over the interval $[0, 1]$. *(3 marks)*

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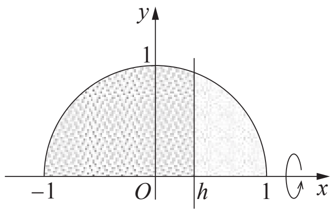
9. Calculus, EXT1 C3 EQ-Bank 23

The region enclosed by the semicircle $y = \sqrt{1 - x^2}$ and the x-axis is to be divided into two pieces by the line $x = h$, when $0 \leq h < 1$.



The two pieces are rotated about the x-axis to form solids of revolution. The value of h is chosen so that the volumes of the solids are in the ratio $2:1$.

Show that h satisfies the equation $3h^3 - 9h + 2 = 0$. *(3 marks)*

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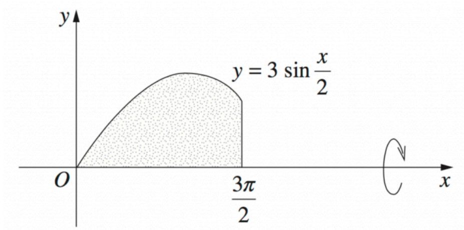
10. Calculus, EXT1 C3 2013 HSC 12b

The region bounded by the graph $y = 3\sin\dfrac{x}{2}$ and the x-axis between $x = 0$ and $x = \dfrac{3\pi}{2}$ is rotated about the x-axis to form a solid.



Find the exact volume of the solid. *(3 marks)*

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11. Calculus, EXT1 C3 2005 HSC 5a

Find the exact value of the volume of the solid of revolution formed when the region bounded by the curve $y = \sin 2x$, the x-axis and the line $x = \dfrac{\pi}{8}$ is rotated about the x-axis. *(3 marks)*

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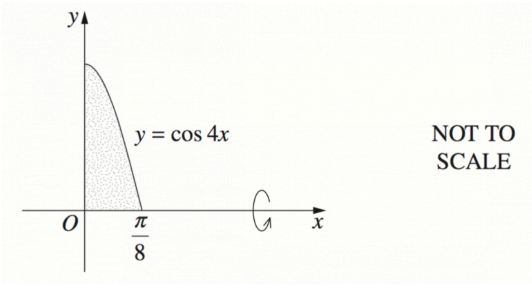
12. Calculus, EXT1 C3 2014 HSC 12b

The region bounded by $y = \cos 4x$ and the x-axis, between $x = 0$ and $x = \frac{\pi}{8}$, is rotated about the x-axis to form a solid.



Find the volume of the solid. *(3 marks)*

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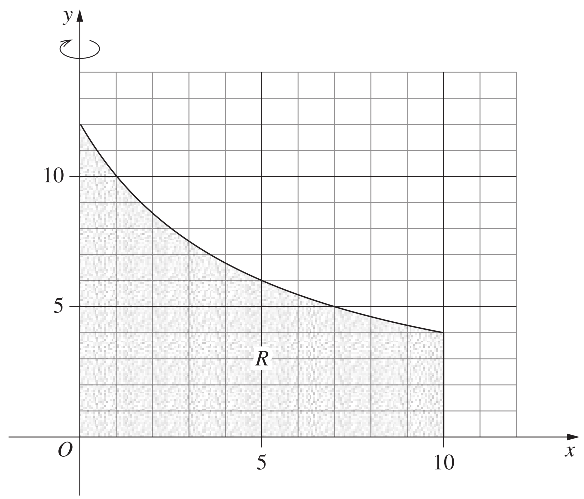
13. Calculus, EXT1 C3 2023 HSC 12e

The region, R, bounded by the hyperbola $y = \dfrac{60}{x+5}$, the line $x = 10$ and the coordinate axes is shown.



Find the volume of the solid of revolution formed when the region R is rotated about the y-axis. Leave your answer in exact form. *(4 marks)*

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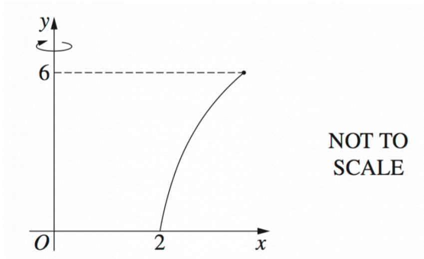
14. Calculus, EXT1* C3 2015 HSC 16b

A bowl is formed by rotating the curve $y = 8\log_e(x-1)$ about the y-axis for $0 \le y \le 6$.



Find the volume of the bowl. Give your answer correct to 1 decimal place. *(3 marks)*

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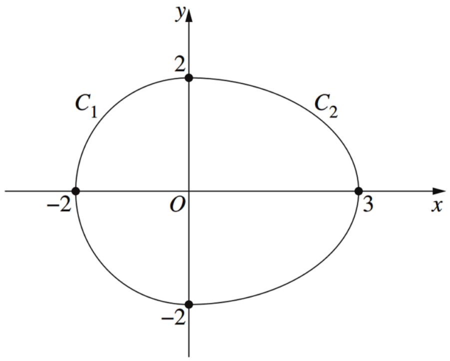
15. Calculus, EXT1* C3 2016 HSC 15a

The diagram shows two curves C_1 and C_2. The curve C_1 is the semicircle $x^2 + y^2 = 4, \quad -2 \le x \le 0$. The curve C_2 has equation $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1, \; 0 \le x \le 3$.



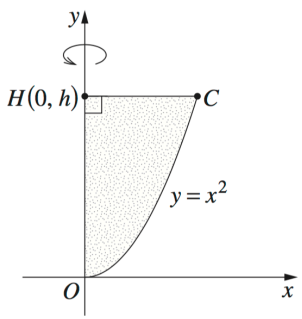
An egg is modelled by rotating the curves about the x-axis to form a solid of revolution.

Find the exact value of the volume of the solid of revolution. ***(4 marks)***

16. Calculus, EXT1* C3 2011 HSC 8b

The diagram shows the region enclosed by the parabola $y = x^2$, the y-axis and the line $y = h$, where $h > 0$. This region is rotated about the y-axis to form a solid called a paraboloid. The point C is the intersection of $y = x^2$ and $y = h$.

The point H has coordinates $(0, h)$.



a. Find the exact volume of the paraboloid in terms of h. ***(2 marks)***

b. A cylinder has radius HC and height h.

What is the ratio of the volume of the paraboloid to the volume of the cylinder? ***(1 mark)***

Worked Solutions

1. Calculus, EXT1 C3 2025 HSC 6 MC

♦ Mean mark 47%.

Since the limits are symmetrical about 0:

Integral will equal zero if function is odd.

Consider option D:

$f(x) = x^2 \tan^{-1}(x)$

$f(-x) = (-x)^2 \tan^{-1}(-x) = -x^2 \tan^{-1}(x) = -f(x)$ (odd)

$\Rightarrow D$

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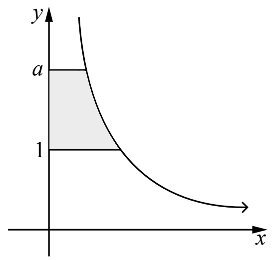
2. Calculus, EXT1 C3 2025 HSC 12b

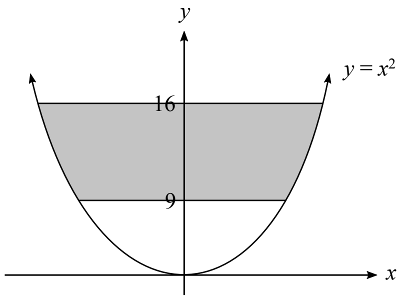


$$y = \frac{1}{x} \;\Rightarrow\; x^2 = \frac{1}{y^2}$$

$$\begin{aligned} V &= \pi \int_1^a x^2\, dy \\ &= \pi \int_1^a y^{-2}\, dy \\ &= -\pi \left[y^{-1}\right]_1^a \\ &= -\pi\left(\frac{1}{a} - 1\right) \\ &= \pi\left(1 - \frac{1}{a}\right) \text{ u}^3 \end{aligned}$$

3. Calculus, EXT1 C3 EQ-Bank 15

a.



b. Areas either side of y-axis are equal.

$$y = x^2 \quad \Rightarrow \quad x = \sqrt{y}$$

$$\begin{aligned} A &= 2\int_9^{16} x \ dy \\ &= 2\int_9^{16} \sqrt{y} \ dy \\ &= 2\left[\frac{2}{3}y^{\frac{3}{2}}\right]_9^{16} \\ &= \frac{4}{3}\left[\left(\sqrt{16}\right)^3 - \left(\sqrt{9}\right)^3\right] \\ &= \frac{4}{3}[64-27] \\ &= \frac{148}{3} \ \text{u}^2 \end{aligned}$$

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4. Calculus, EXT1 C3 2020 HSC 13b

Find intersection:

Mean mark 53%.

$$\begin{aligned} \sin x &= \cos 2x \\ \sin x &= 1 - 2\sin^2 x \end{aligned}$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0$$

$$\sin x = \frac{1}{2} \quad \text{or} \quad \sin x = -1$$

$$x = \frac{\pi}{6} \qquad\qquad x = \frac{3\pi}{2}$$

$$\begin{aligned} V &= \pi\int_0^{\frac{\pi}{6}} (\cos 2x)^2 \ dx - \pi\int_0^{\frac{\pi}{6}} (\sin x)^2 \ dx \\ &= \pi\int_0^{\frac{\pi}{6}} \cos^2 2x - \sin^2 x \ dx \\ &= \pi\int_0^{\frac{\pi}{6}} \frac{1}{2}(1 + \cos 4x) - \frac{1}{2}(1 - \cos 2x) \ dx \\ &= \frac{\pi}{2}\int_0^{\frac{\pi}{6}} \cos 4x + \cos 2x \ dx \\ &= \frac{\pi}{2}\left[\frac{1}{4}\sin 4x + \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{6}} \\ &= \frac{\pi}{8}\left[\sin\frac{2\pi}{3} + 2\sin\frac{\pi}{3}\right] \\ &= \frac{\pi}{8}\left(\frac{\sqrt{3}}{2} + 2 \times \frac{\sqrt{3}}{2}\right) \\ &= \frac{3\sqrt{3}\pi}{16} \ \text{u}^3 \end{aligned}$$

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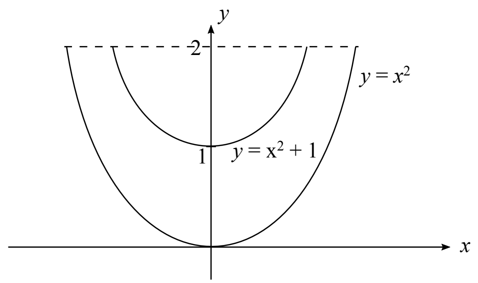
5. Calculus, EXT1 C3 2021 HSC 13a



$$V = \pi\int_0^2 y\ dy - \pi\int_1^2 y - 1\ dy$$

$$= \pi\left[\frac{y^2}{2}\right]_0^2 - \pi\left[\frac{y^2}{2} - y\right]_1^2$$

$$= \pi(2-0) - \pi\left[(2-2) - \left(\frac{1}{2} - 1\right)\right]$$

$$= 2\pi - \pi\left(\frac{1}{2}\right)$$

$$= \frac{3\pi}{2}\ \text{u}^3$$

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6. Calculus, EXT1 C3 2021 HSC 13c

$$A = \text{Area of } \Delta + 2\left|\int_0^2 1 - \frac{8}{4+x^2}\ dx\right|$$

$$= \frac{1}{2}\times 4\times 2 + 2\left|\left[x - 4\tan^{-1}\frac{x}{2}\right]_0^2\right|$$

$$= 4 + 2\left|(2 - 4\tan^{-1}1) - 0\right|$$

$$= 4 + 2\left|2 - \frac{4\pi}{4}\right|$$

$$= 4 + 2(\pi - 2)$$

$$= 2\pi\ \text{u}^2$$

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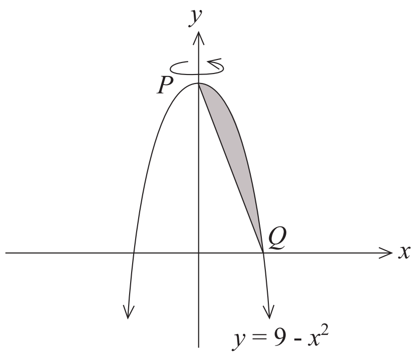
7. Calculus, EXT1 C3 EQ-Bank 21



Equation of PQ:

$m = -3,\quad y\text{-intercept} = 9$

$y = 9 - 3x$

Rotating about the y-axis:

$$x_1^2 = 9 - y \quad \text{(parabola)}$$

$$y = 9 - 3x_2 \quad (\text{line } PQ)$$

$$3x_2 = 9 - y$$

$$x_2 = 3 - \frac{y}{3}$$

$$x_2^2 = \left(3 - \frac{y}{3}\right)^2$$

$$V = \pi\int_0^9 x_1^2 - x_2^2\ dy$$

$$= \pi\int_0^9 9 - y - \left(3 - \frac{y}{3}\right)^2\ dy$$

$$= \pi\int_0^9 9 - y - \left(9 - 2y + \frac{y^2}{9}\right)\ dy$$

$$= \pi\int_0^9 y - \frac{y^2}{9}\ dy$$

$$= \pi\left[\frac{y^2}{2} - \frac{y^3}{27}\right]_0^9$$

$$= \pi\left(\frac{81}{2} - 27\right)$$

$$= \frac{27\pi}{2} \text{ units}^3$$

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8. Calculus, EXT1 C3 EQ-Bank 22

$$V = \pi \int_0^1 \frac{1+2x}{1+x^2}\ dx$$

$$= \pi \int_0^1 \frac{1}{1+x^2}\ dx + \pi \int_0^1 \frac{2x}{1+x^2}\ dx$$

$$= \pi \left[\tan^{-1}(x)\right]_0^1 + \pi \left[\ln\left(1+x^2\right)\right]_0^1$$

$$= \pi\left(\tan^{-1}1 - \tan^{-1}0\right) + \pi(\ln 2 - \ln 1)$$

$$= \pi\left(\frac{\pi}{4}\right) + \pi(\ln 2)$$

$$= \pi\left(\frac{\pi}{4} + \ln 2\right) \ \text{ u}^3$$

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9. Calculus, EXT1 C3 EQ-Bank 23

Volume of smaller solid

$$= \pi \int_h^1 \left(\sqrt{1-x^2}\right)^2\ dx$$

$$= \pi \int_h^1 1 - x^2\ dx$$

$$= \pi \left[x - \frac{x^3}{3}\right]_h^1$$

$$= \pi \left[\left(1 - \frac{1}{3}\right) - \left(h - \frac{h^3}{3}\right)\right]$$

$$= \pi \left(\frac{2}{3} - h + \frac{h^3}{3}\right)$$

Since smaller solid is $\frac{1}{3}$ volume of sphere:

$$\pi\left(\frac{2}{3} - h + \frac{h^3}{3}\right) = \frac{1}{3} \times \frac{4}{3} \cdot \pi \cdot 1^3$$

$$\frac{h^3}{3} - h + \frac{2}{3} = \frac{4}{9}$$

$$3h^3 - 9h + 6 = 4$$

$$\therefore 3h^3 - 9h + 2 = 0 \quad \ldots \text{as required}$$

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10. Calculus, EXT1 C3 2013 HSC 12b

$$y = 3\sin\ \frac{x}{2}$$
$$y^2 = 9\sin^2\ \frac{x}{2}$$

$$\text{Using: } \sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\begin{aligned}
\therefore V &= \pi \int_0^{\frac{3\pi}{2}} 9\sin^2\ \frac{x}{2}\ dx \\
&= \frac{9\pi}{2} \int_0^{\frac{3\pi}{2}} (1 - \cos x)\ dx \\
&= \frac{9\pi}{2} [x - \sin x]_0^{\frac{3\pi}{2}} \\
&= \frac{9\pi}{2} \left[\left(\frac{3\pi}{2} - \sin\ \frac{3\pi}{2} \right) - 0 \right] \\
&= \frac{9\pi}{2} \left(\frac{3\pi}{2} + 1 \right)\ \text{u}^3
\end{aligned}$$

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11. Calculus, EXT1 C3 2005 HSC 5a

$$y = \sin 2x$$
$$y^2 = \sin^2 2x$$

$$\text{Using: } \sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

> **COMMENT:** Michael Wells (1st in state Ext2) would derive this formula in his working from the $\sin^2 x + \cos^2 = 1$ identity in ~5 seconds *every time* he used it in an exam.

$$\begin{aligned}
\therefore V &= \pi \int_0^{\frac{\pi}{8}} y^2\ dx \\
&= \pi \int_0^{\frac{\pi}{8}} \sin^2 2x\ dx \\
&= \frac{\pi}{2} \int_0^{\frac{\pi}{8}} 1 - \cos\ 4x\ dx \\
&= \frac{\pi}{2} \left[x - \frac{1}{4}\sin\ 4x \right]_0^{\frac{\pi}{8}} \\
&= \frac{\pi}{2} \left[\left(\frac{\pi}{8} - \frac{1}{4}\sin\ \frac{\pi}{2} \right) - 0 \right] \\
&= \frac{\pi}{2} \left(\frac{\pi}{8} - \frac{1}{4} \right) \\
&= \frac{\pi}{2} \left(\frac{\pi - 2}{8} \right) \\
&= \frac{\pi}{16}(\pi - 2)\quad \text{u}^3
\end{aligned}$$

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12. Calculus, EXT1 C3 2014 HSC 12b

$$\begin{aligned}
V &= \pi\int_0^{\frac{\pi}{8}} y^2\ dx \\
&= \pi\int_0^{\frac{\pi}{8}} \cos^2 4x\ dx \\
&= \pi\int_0^{\frac{\pi}{8}} \frac{1}{2}(\cos 8x + 1)\ dx \\
&= \frac{\pi}{2}\left[\frac{1}{8}\sin 8x + x\right]_0^{\frac{\pi}{8}} \\
&= \frac{\pi}{2}\left[\left(\frac{1}{8}\sin\pi + \frac{\pi}{8}\right)\right]-0\Big] \\
&= \frac{\pi^2}{16}\ \text{u}^3
\end{aligned}$$

COMMENT: The identities $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$ are tested every year – *know them.*

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13. Calculus, EXT1 C3 2023 HSC 12e

$$y = \frac{60}{x+5} \quad \Rightarrow \quad x = \frac{60}{y} - 5$$

♦ Mean mark 49%.

$$\begin{aligned}
V &= \pi\int_4^{12} x^2\ dy + \pi r^2 h \\
&= \pi\int_4^{12} \left(\frac{60}{y} - 5\right)^2 dy + \pi \times 10^2 \times 4 \\
&= 25\pi\int_4^{12} \left(\frac{12}{y} - 1\right)^2 dy + 400\pi \\
&= 25\pi\int_4^{12} \left(\frac{144}{y^2} - \frac{24}{y} + 1\right) dy + 400\pi \\
&= 25\pi\left[\frac{-144}{y} - 24\ln y + y\right]_4^{12} + 400\pi \\
&= 25\pi\Big[(-12 - 24\ln 12 + 12) - (-36 - 24\ln 4 + 4)\Big] + 400\pi \\
&= 25\pi(24\ln 4 - 24\ln 12 + 32) + 400\pi \\
&= 600\pi\ln\left(3^{-1}\right) + 800\pi + 400\pi \\
&= 1200\pi - 600\pi\ln 3\ \ \text{u}^3
\end{aligned}$$

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14. Calculus, EXT1* C3 2015 HSC 16b

♦ *Mean mark 38%.*

$$\begin{aligned}
y &= 8\log_e(x-1) \\
\frac{y}{8} &= \log_e(x-1) \\
e^{\frac{y}{8}} &= x-1 \\
x &= e^{\frac{y}{8}} + 1 \\
x^2 &= \left(e^{\frac{y}{8}} + 1\right)^2 \\
&= \left(e^{\frac{y}{8}}\right)^2 + 2e^{\frac{y}{8}} + 1 \\
&= e^{\frac{y}{4}} + 2e^{\frac{y}{8}} + 1
\end{aligned}$$

$$\begin{aligned}
V &= \pi\int_0^6 x^2\ dy \\
&= \pi\int_0^6 \left(e^{\frac{y}{4}} + 2e^{\frac{y}{8}} + 1\right)\ dy \\
&= \pi\left[4e^{\frac{y}{4}} + 16e^{\frac{y}{8}} + y\right]_0^6 \\
&= \pi\left[\left(4e^{\frac{6}{4}} + 16e^{\frac{6}{8}} + 6\right) - \left(4e^0 + 16e^0 + 0\right)\right] \\
&= \pi\left[\left(4e^{1.5} + 16e^{0.75} + 6\right) - (4 + 16)\right] \\
&= \pi\left[4e^{1.5} + 16e^{0.75} - 14\right] \\
&= 118.748\ldots \\
&= 118.7\ \ \text{(to 1 d.p.)}
\end{aligned}$$

$\therefore$ Volume of the bowl is 118.7 u³.

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15. Calculus, EXT1* C3 2016 HSC 15a

Consider C_1,

$$\begin{aligned} V_1 &= \pi \int_{-2}^{0} y^2\ dx \\ &= \pi \int_{-2}^{0} 4 - x^2\ dx \\ &= \pi \left[4x - \frac{x^3}{3} \right]_{-2}^{0} \\ &= \pi \left[0 - \left(-8 + \frac{8}{3} \right) \right] \\ &= \frac{16\pi}{3}\ u^3 \end{aligned}$$

Consider C_2

$$\begin{aligned} \frac{x^2}{9} + \frac{y^2}{4} &= 1 \\ y^2 &= 4 - \frac{4x^2}{9} \end{aligned}$$

$$\begin{aligned} V_2 &= \pi \int_{0}^{3} 4 - \frac{4x^2}{9}\ dx \\ &= \pi \left[4x - \frac{4x^3}{27} \right]_{0}^{3} \\ &= \pi \left[\left(12 - \frac{4 \cdot 3^3}{27} \right) - 0 \right] \\ &= 8\pi\ \text{u}^3 \end{aligned}$$

$$\begin{aligned} \text{Volume} &= V_1 + V_2 \\ &= \frac{16\pi}{3} + 8\pi \\ &= \frac{40\pi}{3}\ \text{u}^3 \end{aligned}$$

16. Calculus, EXT1* C3 2011 HSC 8b

IMPORTANT: Most common errors: *1-use the correct axis, and 2-check the limits!*

a.
$$\begin{aligned} V &= \pi \int_{0}^{h} x^2\ dy \\ &= \pi \int_{0}^{h} y\ dy \\ &= \pi \left[\frac{1}{2} y^2 \right]_{0}^{h} \\ &= \pi \left(\frac{1}{2} h^2 \right) \\ &= \frac{\pi h^2}{2}\ \text{u}^3 \end{aligned}$$

$\therefore$ The volume of the paraboloid is $\dfrac{\pi h^2}{2}$ u^3

b. Radius of cylinder $(r) = HC$

Find x-coordinate of C:

♦♦ Mean mark of 24%.

$$\begin{aligned} \text{When } y &= h \\ \Rightarrow x^2 &= h \\ x &= \sqrt{h} \\ \therefore r &= \sqrt{h} \end{aligned}$$

$$\begin{aligned} \text{Volume of cylinder} &= \pi r^2 h \\ &= \pi \left(\sqrt{h} \right)^2 h \\ &= \pi h^2 \end{aligned}$$

$\therefore$ Volume of paraboloid : volume of cylinder

$$\begin{aligned} &= \frac{\pi h^2}{2} : \pi h^2 \\ &= 1 : 2 \end{aligned}$$