MATHEMATICS ADVANCED

SmarterEd Maths & Sciences

HSC REVISION SERIES 2026

Welcome to the SmarterMaths 2026 HSC Advanced Revision Series.

Key features of this year's Revision Series include:

- ~22 hours of carefully selected HSC revision questions
- Targeted practice across the most important HSC question types
- Designed for students aiming for a Band 5-6 result
- Higher weighting towards historically difficult question types
- Topic and question weightings reflect historical HSC exam allocations
- Build your own personalised revision resource by ***carefully reviewing and annotating*** these worksheets throughout Term 3.

**Philosophy: Every question has earned its place. Historical HSC trends, common question types and past areas of student difficulty have all influenced the selection process.*

IMPORTANT

If you've already completed some of these questions during the year, **that's a good thing.**

Many of the highest-performing students credit repeated practice with challenging past HSC questions as a key ingredient in their HSC success. This revision series focuses on the highest-value question types, helping you build confidence and critically, *speed* through the exam.

HSC Final Study – Advanced Maths: Trigonometry

Worksheet 2 of 2 | Estimated exam weighting: 15.3%

Syllabus: T2 Trig Functions and Identities, T3 Trig Functions and Graphs

Key Areas addressed by this worksheet:

<u>T2 Trig Functions and Identities (1.8%)</u>

- A small contributor that remains important due to the critical gateway knowledge it covers for many later topics.
- **Exact Trig Ratios**: attracted a 3-mark question in both 2025 and 2023 after being absent 2019-22. Note the use of "dedicated" here as this concept is tested multiple times every year as part of applied topics covered later in the revision series.
- The most common question type requires students to solve a simple trig equation in exact radian form within a specified range. Numerous examples look at this, including the harder *2008 HSC 6a* which caused problems.
- *2025 Adv 31* was a beast in this topic area that deserves specific attention.
- Questions giving an exact trig ratio that require students to use Pythagoras and find exact *sec, cosec* and *cot* ratios is covered (similar to NESA Topic Guidance exemplar questions).
- **Trig Identities and Harder Equations:** examined in 2025 and 2020-21.
- This topic can be examined in a broad array of questions that are often challenging. The revision set features multiple harder examples.

<u>T3 Trig Functions and Graphs (6.2%)</u>

- **Trig Graphs (★):** examined in each new syllabus exam between 2020-22 and notably absent in the last three years.
- *Trig Graphs* will involve transformations and require students to sketch or recognize these graphs. Multiple examples cover this area.
- Amplitude and period of various trig functions is well covered (including the period of *tan* functions which has caused problems in the past).
- We also revise examples that reflect NESA sample exam questions in this topic area.
- **Trig Applications (★):** examined in 2020, 2022 and 2024. When this content is examined, it typically comes with highly significant mark allocations. The need for a revision focus here is a statement of the bleeding obvious.

KEY: ★ Priority Revision – content or specific questions flagged by SM analysis as key revision.

ADVANCED 2026

HSC Revision Series

Smarter Ed

Trigonometry (2 of 2)

T2 Trig Functions and Identities (Y11)

T3 Trig Functions and Graphs (Y12)

Exam Equivalent Time: 90 minutes *(based on allocation of 1.5 minutes per mark)*

Questions

1. Trigonometry, 2ADV T3 EQ-Bank 2 MC

Let $f(x) = 1 - 2\cos\left(\dfrac{\pi x}{2}\right)$.

The period and range of this function are respectively

A. 4 and $[-2, 2]$

B. 4 and $[-1, 3]$

C. 1 and $[-1, 3]$

D. 4π and $[-2, 2]$

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2. Trigonometry, 2ADV T2 2012 HSC 6 MC

What are the solutions of $\sqrt{3}\tan x = -1$ for $0 \leq x \leq 2\pi$?

A. $\dfrac{2\pi}{3}$ and $\dfrac{4\pi}{3}$

B. $\dfrac{2\pi}{3}$ and $\dfrac{5\pi}{3}$

C. $\dfrac{5\pi}{6}$ and $\dfrac{7\pi}{6}$

D. $\dfrac{5\pi}{6}$ and $\dfrac{11\pi}{6}$

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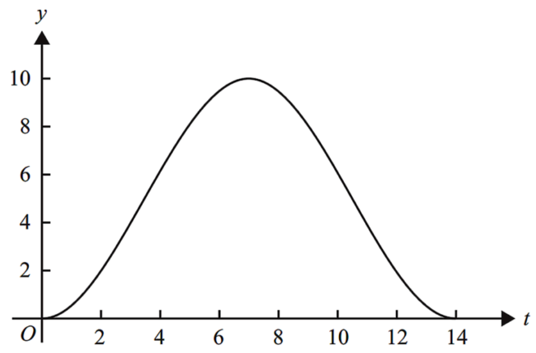
3. Trigonometry, 2ADV T3 EQ-Bank 5 MC

The UV index, y, for a summer day in Newcastle East is illustrated in the graph below, where t is the number of hours after 6 am.



The graph is most likely to be the graph of

A. $y = 5 + 5\cos\left(\dfrac{\pi t}{7}\right)$

B. $y = 5 - 5\cos\left(\dfrac{\pi t}{7}\right)$

C. $y = 5 + 5\cos\left(\dfrac{\pi t}{14}\right)$

D. $y = 5 - 5\cos\left(\dfrac{\pi t}{14}\right)$

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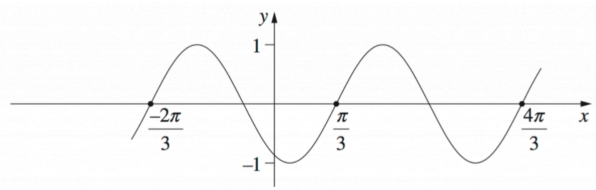
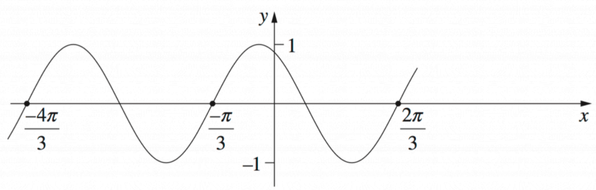
4. Trigonometry, 2ADV T3 2013 HSC 6 MC

Which diagram shows the graph $y = \sin\left(2x + \frac{\pi}{3}\right)$?

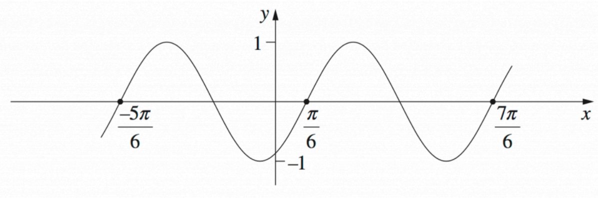
(A)



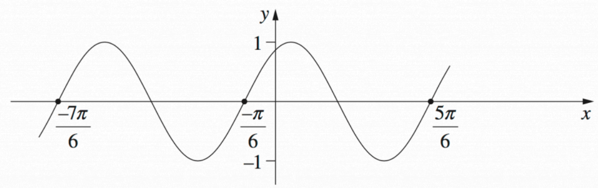
(B)



(C)



(D)



5. Trigonometry, 2ADV T3 2016 HSC 6 MC

What is the period of the function $f(x) = \tan(3x)$?

A. $\frac{\pi}{3}$

B. $\frac{2\pi}{3}$

C. 3π

D. 6π

6. Trigonometry, 2ADV T2 2014 HSC 7 MC

How many solutions of the equation $(\sin x - 1)(\tan x + 2) = 0$ lie between 0 and 2π?

A. 1

B. 2

C. 3

D. 4

7. Trigonometry, 2ADV T2 EQ-Bank 18

Given $\cot\theta = -\frac{24}{7}$ for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, find the exact value of

a. $\sec\theta$ *(2 marks)*

b. $\sin\theta$ *(1 mark)*

8. Trigonometry, 2ADV T2 EQ-Bank 23

Solve the equation $\cos\left(\frac{3x}{2}\right) = \frac{1}{2}$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$. *(2 marks)*

9. Trigonometry, 2ADV T2 EQ-Bank 25

Solve the equation

$\sin\left(2x + \frac{\pi}{3}\right) = \frac{1}{2}$ for $0 \leq x \leq \pi$ *(2 marks)*

10. Trigonometry, 2ADV T2 EQ-Bank 26

Solve the equation $2\cos 2\theta = 2\sin 2\theta$ for $0 \leqslant \theta \leqslant 2\pi$ *(2 marks)*

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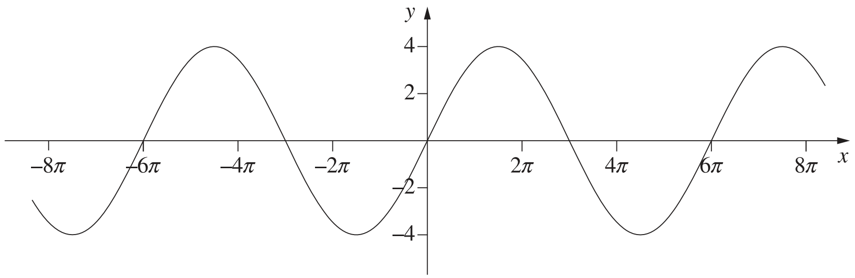
11. Trigonometry, 2ADV T3 2022 HSC 14

The graph of $y = k\sin(ax)$



What are the values of k and a? *(2 marks)*

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12. Trigonometry, 2ADV T3 EQ-Bank 23

State the range and period of the function

$$h(x) = 4 + 3\cos\left(\frac{\pi x}{2}\right).$$ *(2 marks)*

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13. Trigonometry, 2ADV T2 2023 HSC 20

Find all the values of θ, where $0^\circ \leq \theta \leq 360^\circ$, such that

$$\sin(\theta - 60^\circ) = -\frac{\sqrt{3}}{2}$$ *(3 marks)*

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14. Trigonometry, 2ADV T3 EQ-Bank 24

The graphs of $y = \cos(x)$ and $y = a\sin(x)$, where a is a real constant, have a point of intersection at $x = \dfrac{\pi}{3}$.

a. Find the value of a. *(2 marks)*

b. Find the x-coordinate of the other point of intersection of the two graphs, given $0 \leq x \leq 2\pi$ *(1 mark)*

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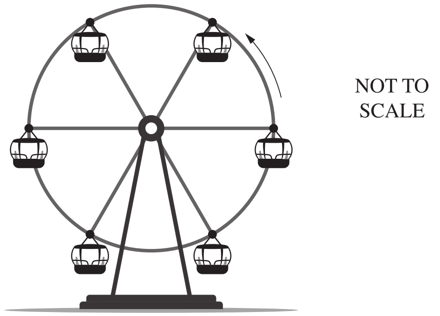
15. Trigonometry, 2ADV T3 2024 HSC 28

Anna is sitting in a carriage of a Ferris wheel which is revolving. The height, $A(t)$, in metres above the ground of the top of her carriage is given by

$$A(t) = c - k\cos\left(\frac{\pi t}{24}\right),$$

where t is the time in seconds after Anna's carriage first reaches the bottom of its revolution and c and k are constants.



The top of each carriage reaches a greatest height of 39 metres and a smallest height of 3 metres.

a. Find the value of c and k. *(2 marks)*

b. How many seconds does it take for one complete revolution of the Ferris wheel? *(1 mark)*

c. Billie is in another carriage. The height, $B(t)$, in metres above the ground of the top of her carriage is given by

$$B(t) = c - k\cos\left(\frac{\pi}{24}(t - 6)\right),$$

where c and k are as found in part (a).

During each revolution, there are two occasions when Anna's and Billie's carriages are at the same heights. At what two heights does this occur? Give your answer correct to 2 decimal places. *(4 marks)*

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16. Trigonometry, 2ADV T3 2022 HSC 23

The depth of water in a bay rises and falls with the tide. On a particular day the depth of the water, d metres, can be modelled by the equation

$$d = 1.3 - 0.6\cos\left(\frac{4\pi}{25}t\right),$$

where t is the time in hours since low tide.

a. Find the depth of water at low tide and at high tide. *(2 marks)*

b. What is the time interval, in hours, between two successive low tides? *(1 mark)*

c. For how long between successive low tides will the depth of water be at least 1 metre? *(3 marks)*

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17. Trigonometry, 2ADV T2 2014 HSC 15a

Find all solutions of $2\sin^2 x + \cos x - 2 = 0$, where $0 \leq x \leq 2\pi$. *(3 marks)*

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18. Trigonometry, 2ADV T2 2019 HSC 13a

Solve $2\sin x\cos x = \sin x$ for $0 \leq x \leq 2\pi$. *(3 marks)*

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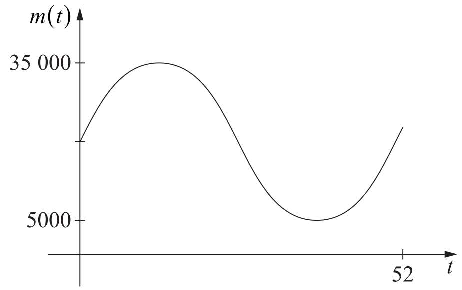
19. Trigonometry, 2ADV T3 2020 HSC 31

The population of mice on an isolated island can be modelled by the function.

$$m(t) = a\sin\left(\frac{\pi}{26}t\right) + b,$$

where t is the time in weeks and $0 \leq t \leq 52$. The population of mice reaches a maximum of 35 000 when $t = 13$ and a minimum of 5000 when $t = 39$. The graph of $m(t)$ is shown.



a. What are the values of a and b? *(2 marks)*

b. On the same island, the population of cats can be modelled by the function

$$c(t) = -80\cos\left(\frac{\pi}{26}(t - 10)\right) + 120$$

Consider the graph of $m(t)$ and the graph of $c(t)$.

Find the values of t, $0 \leq t \leq 52$, for which both populations are increasing. *(3 marks)*

c. Find the rate of change of the mice population when the cat population reaches a maximum. *(2 marks)*

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20. Trigonometry, 2ADV T3 2009 HSC 7b

Between 5 am and 5 pm on 3 March 2009, the height, h, of the tide in a harbour was given by

$$h = 1 + 0.7\sin\left(\frac{\pi}{6}t\right) \quad \text{for} \quad 0 \leq t \leq 12$$

where h is in metres and t is in hours, with $t = 0$ at 5 am.

a. What is the period of the function h? *(1 mark)*

b. What was the value of h at low tide, and at what time did low tide occur? *(2 marks)*

c. A ship is able to enter the harbour only if the height of the tide is at least 1.35 m.

Find all times between 5 am and 5 pm on 3 March 2009 during which the ship was able to enter the harbour. *(3 marks)*

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21. Trigonometry, 2ADV T2 2025 HSC 31

The equation $\cos px = \dfrac{1}{2}$ has 2 solutions where $0 \leq x \leq 2\pi$ and $p > 0$.

Find all possible values of p. *(3 marks)*

Worked Solutions

1. Trigonometry, 2ADV T3 EQ-Bank 2 MC

$$\text{Period} = \frac{2\pi}{n} = \frac{2\pi}{\frac{\pi}{2}} = 4$$

$$\text{Amplitude} = 2$$

Graph centre line (median): $y = 1$.

$$\therefore \text{ Range} = [1-2,\ 1+2]$$
$$= [-1, 3]$$

$\Rightarrow B$

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2. Trigonometry, 2ADV T2 2012 HSC 6 MC

$$\sqrt{3}\tan x = -1$$

$$\tan x = -\frac{1}{\sqrt{3}}$$

$$\text{When } \tan x = \frac{1}{\sqrt{3}}, \quad x = \frac{\pi}{6}$$

Since $\tan x$ is negative in $2^{\text{nd}}/4^{\text{th}}$ quadrant

$$\therefore x = \pi - \frac{\pi}{6},\ 2\pi - \frac{\pi}{6},\ \ldots$$

$$= \frac{5\pi}{6},\ \frac{11\pi}{6}$$

$\Rightarrow D$

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3. Trigonometry, 2ADV T3 EQ-Bank 5 MC

Centre line (median): $y = 5$

Amplitude $= 5$

$$\text{Period:}\ \ 14 = \frac{2\pi}{n}$$

$$n = \frac{\pi}{7}$$

$$\therefore\ \text{Graph:}\ \ y = 5 - 5\cos\left(\frac{\pi t}{7}\right)$$

$\Rightarrow\ B$

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4. Trigonometry, 2ADV T3 2013 HSC 6 MC

$$\text{At}\ x = 0,\ \ y = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

♦♦ *Mean mark 34%*

$\Rightarrow$ It cannot be A or C

Find x when $y = 0$,

$$\sin\left(2x + \frac{\pi}{3}\right) = 0$$

$$\therefore\ 2x + \frac{\pi}{3} = 0 \qquad (\sin 0 = 0)$$

$$2x = -\frac{\pi}{3}$$

$$x = -\frac{\pi}{6}$$

$\Rightarrow\ D$

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5. Trigonometry, 2ADV T3 2016 HSC 6 MC

$$\text{Period} = \frac{\pi}{n}$$

$$= \frac{\pi}{3}$$

♦ *Mean mark 42%.*

$\Rightarrow\ A$

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6. Trigonometry, 2ADV T2 2014 HSC 7 MC

When $(\sin x - 1)(\tan x + 2) = 0$

$(\sin x - 1) = 0$ or $\tan x + 2 = 0$

If $\sin x - 1 = 0$:

$$\sin x = 1 \ \ \Rightarrow \ \ x = \frac{\pi}{2}, \quad 0 < x < 2\pi$$

If $\tan x + 2 = 0$:

$\tan x = -2$

Since $\tan \dfrac{\pi}{2}$ is undefined, there are only 2 solutions when

$\tan x = -2$ (which occurs in the 1st and 4th quadrants).

$\therefore$ 2 solutions

$\Rightarrow\ B$

♦♦♦ *Mean mark 25%, making it the toughest MC question in the 2014 exam.*

COMMENT: Note that the "2 solutions" answer relies on the sum of an infinity of zeros not equalling zero. This concept created unintended difficulty in this question.

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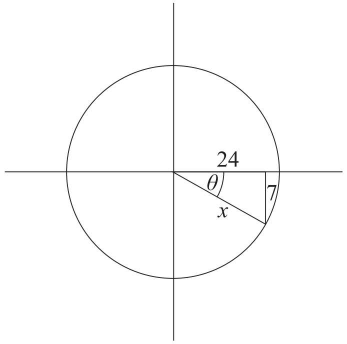
7. Trigonometry, 2ADV T2 EQ-Bank 18

a. $\cot\theta = -\dfrac{24}{7} \quad \Rightarrow \quad \tan\theta = -\dfrac{7}{24}$

Graphically, given $-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$:



$$x = \sqrt{24^2 + 7^2} = 25$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{24}{25}} = \frac{25}{24}$$

b. $\sin\theta = -\dfrac{7}{25}$

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8. Trigonometry, 2ADV T2 EQ-Bank 23

$$\cos\left(\frac{3x}{2}\right) = \frac{1}{2}$$

$$\Rightarrow \text{ Base angle } = \frac{\pi}{3}$$

$$\frac{3x}{2} = \frac{-\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3}, \ldots$$

$$\therefore x = \frac{-2\pi}{9}, \frac{2\pi}{9}, \frac{10\pi}{9}$$

$$= \frac{-2\pi}{9}, \frac{2\pi}{9} \quad \left(-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}\right)$$

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9. Trigonometry, 2ADV T2 EQ-Bank 25

$$\sin\left(2x + \frac{\pi}{3}\right) = \frac{1}{2}$$

$$\Rightarrow \text{ Base angle is } \frac{\pi}{6}$$

$$\left(2x + \frac{\pi}{3}\right) = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \ldots$$

$$2x = -\frac{\pi}{6}, \frac{\pi}{2}, \frac{11\pi}{6}, \frac{15\pi}{6}, \ldots$$

$$x = -\frac{\pi}{12}, \frac{\pi}{4}, \frac{11\pi}{12}, \frac{15\pi}{12}, \ldots$$

$$\therefore x = \frac{\pi}{4}, \frac{11\pi}{12} \quad (0 \leq x \leq \pi)$$

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10. Trigonometry, 2ADV T2 EQ-Bank 26

$$2\cos 2\theta = 2\sin 2\theta$$

$$\frac{2\sin 2\theta}{2\cos 2\theta} = 1$$

$$\tan 2\theta = 1$$

$$2\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}, \ldots$$

$$\theta = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8} \quad (0 \leqslant \theta \leqslant 2\pi)$$

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11. Trigonometry, 2ADV T3 2022 HSC 14

$$\text{Amplitude} = 4$$

$$\Rightarrow k = 4$$

$$\text{Period} = 6\pi$$

$$\frac{2\pi}{a} = 6\pi$$

$$6\pi a = 2\pi$$

$$\Rightarrow a = \frac{1}{3}$$

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12. Trigonometry, 2ADV T3 EQ-Bank 23

$$-1 \le \cos\left(\frac{\pi x}{2}\right) \le 1$$
$$-3 \le 3\cos\left(\frac{\pi x}{2}\right) \le 3$$
$$1 \le 4 + 3\cos\left(\frac{\pi x}{2}\right) \le 7$$

$\therefore$ Range: $1 \le y \le 7$

$$\text{Period } = \frac{2\pi}{n} = \frac{2\pi}{\frac{\pi}{2}} = 4$$

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13. Trigonometry, 2ADV T2 2023 HSC 20

$\sin 60^\circ = \dfrac{\sqrt{3}}{2} \Rightarrow$ Base angle $= 60^\circ$

$\Rightarrow$ sin is negative in 3rd and 4th quadrants

$$\sin(\theta - 60^\circ) = 180 + 60, 360 - 60$$
$$= 240^\circ, 300^\circ$$

$\theta - 60^\circ = 240^\circ \Rightarrow \theta = 300^\circ$

$\theta - 60^\circ = 300^\circ \Rightarrow \theta = 360^\circ$

Consider $\theta = 0^\circ$

$$\sin(0 - 60^\circ) = \sin(-60^\circ) = -\frac{\sqrt{3}}{2}$$

$\therefore \theta = 0^\circ, 300^\circ$ and 360°

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14. Trigonometry, 2ADV T3 EQ-Bank 24

a. Intersection occurs when $x = \dfrac{\pi}{3}$,

$$a\sin\left(\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right)$$
$$\tan\left(\frac{\pi}{3}\right) = \frac{1}{a}$$
$$\sqrt{3} = \frac{1}{a}$$
$$\therefore a = \frac{1}{\sqrt{3}}$$

b. $\tan(x) = \sqrt{3}$

$$x = \frac{\pi}{3}, \frac{4\pi}{3}, 2\pi + \frac{\pi}{3}, \ldots$$
$$\therefore x = \frac{4\pi}{3} \quad (0 \le x \le 2\pi)$$

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15. Trigonometry, 2ADV T3 2024 HSC 28

a. $c =$ centre of motion $= \dfrac{39+3}{2} = 21$

$k =$ amplitude $= \dfrac{39-3}{2} = 18$

b. $A(t) = 21 - 18\cos\left(\dfrac{\pi t}{24}\right) \Rightarrow n = \dfrac{\pi}{24}$

$T = \dfrac{2\pi}{n} = 2\pi \times \dfrac{24}{\pi} = 48$ seconds

c. Strategy 1

♦♦ Mean mark (c) 38%.

Billie's carriage is 6 seconds behind Anna's.

When $t = 0$, Anna's carriage is at the lowest point $= 21 - 18 = 3$

When $t = 3$, by symmetry, both carriages are at the same height:

$$h_1 = 21 - 18\cos\left(\frac{\pi}{8}\right) = 4.370\ldots = 4.37 \text{ m (2 d.p.)}$$

When $t = 24$, Anna's carriage is at the highest point $= 21 + 18 = 39$

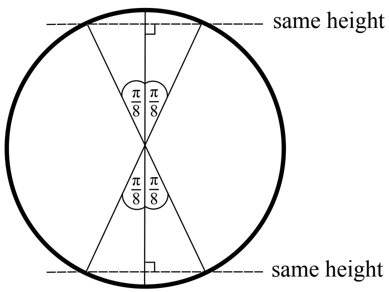
When $t = 27$, by symmetry, both carriages are at the same height:

$$h_2 = 21 - 18\cos\left(\frac{9\pi}{8}\right) = 37.629\ldots = 37.63 \text{ m (2 d.p.)}$$

Strategy 2

Angle between the 2 carriages $= \dfrac{\pi \times 6}{24} = \dfrac{\pi}{4}$

By inspection:



Heights are the same:

$$h_1 = 21 - 18\cos\left(\frac{\pi}{8}\right) = 4.370\ldots = 4.37 \text{ m (2 d.p.)}$$

$$h_2 = 21 - 18\cos\left(\frac{7\pi}{8}\right) = 37.629\ldots = 37.63 \text{ m (2 d.p.)}$$

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16. Trigonometry, 2ADV T3 2022 HSC 23

a. Since $-1 \leq \cos\left(\dfrac{4\pi}{25}t\right) \leq 1$:

Low Tide $= 1.3 - 0.6(1) = 0.7$ m

High Tide $= 1.3 - 0.6(-1) = 1.9$ m

b. Time between two low tides = Period of equation (n)

♦ Mean mark (b) 47%.

$$\frac{2\pi}{n} = \frac{4\pi}{25}$$

$$\frac{n}{2\pi} = \frac{25}{4\pi}$$

$$n = \frac{25}{2} \text{ hours}$$

c. Find t when $d = 1$:

♦ Mean mark (c) 46%.

$$1.3 - 0.6\cos\left(\frac{4\pi}{25}t\right) = 1$$

$$-0.6\cos\left(\frac{4\pi}{25}t\right) = -0.3$$

$$\cos\left(\frac{4\pi}{25}t\right) = \frac{1}{2}$$

$$\frac{4\pi}{25}t = \frac{\pi}{3}, \ \frac{5\pi}{3}$$

$$t = \frac{25}{12}, \ \frac{125}{12}$$

$\therefore$ Time between low tides where water depth ≥ 1 m

$$= \frac{125}{12} - \frac{25}{12} = \frac{100}{12} = \frac{25}{3} \text{ hours}$$

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17. Trigonometry, 2ADV T2 2014 HSC 15a

$$2\sin^2 x + \cos x - 2 = 0$$
$$2\left(1 - \cos^2 x\right) + \cos x - 2 = 0$$
$$2 - 2\cos^2 x + \cos x - 2 = 0$$
$$-2\cos^2 x + \cos x = 0$$
$$\cos x(-2\cos x + 1) = 0$$

♦ *Mean mark 42%*

$\therefore -2\cos x + 1 = 0$ or $\cos x = 0$

$2\cos x = 1 \qquad x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$

$\cos x = \dfrac{1}{2}$

$\cos\left(\dfrac{\pi}{3}\right) = \dfrac{1}{2}$

Since cos is positive in $1^{\text{st}}/4^{\text{th}}$ quadrants:

$x = \dfrac{\pi}{3}, 2\pi - \dfrac{\pi}{3} = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$

$\therefore x = \dfrac{\pi}{3}, \dfrac{\pi}{2}, \dfrac{3\pi}{2}, \dfrac{5\pi}{3}$ for $0 \le x \le 2\pi$

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18. Trigonometry, 2ADV T2 2019 HSC 13a

$2\sin x\cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

♦ *Mean mark 49%.*

$\sin x = 0$

$\Rightarrow x = 0, \pi, 2\pi$

$\cos x = \dfrac{1}{2}$

$\Rightarrow x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$

$\therefore x = 0, \dfrac{\pi}{3}, \pi, \dfrac{5\pi}{3}, 2\pi$

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19. Trigonometry, 2ADV T3 2020 HSC 31

a. $b = \dfrac{35\ 000 + 5000}{2} = 20\ 000$

$a =$ amplitude of sin graph

$= 35\ 000 - 20\ 000$

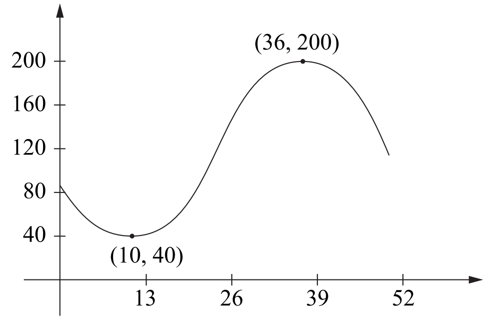
$= 15\ 000$

b. By inspection of the $m(t)$ graph

$m'(t) > 0$ when $0 \le t < 13$ and $39 < t \le 52$

♦♦ *Mean mark (b) 30%.*

Sketch $c(t)$:



Minimum $(\cos 0)$ when $t = 10$

Maximum $(\cos\pi)$ when $t = 36$

$\therefore c'(t) > 0$ when $10 < t < 36$

$\therefore$ Both populations are increasing when $10 < t < 13$

c. $c(t)$ maximum when $t = 36$

$$m(t) = 15\ 000\sin\left(\frac{\pi}{26}t\right) + 20\ 000$$

♦♦♦ *Mean mark (c) 27%.*

$$m'(t) = \frac{15\ 000\pi}{26}\cos\left(\frac{\pi}{26}t\right)$$
$$m'(36) = \frac{15\ 000\pi}{26}\cdot\cos\left(\frac{36\pi}{26}\right)$$
$$= -642.7$$

$\therefore$ Mice population is decreasing at 643 mice per week.

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20. Trigonometry, 2ADV T3 2009 HSC 7b

a. $h = 1 + 0.7\sin\left(\frac{\pi}{6}t\right)$ for $0 \leq t \leq 12$

$$T = \frac{2\pi}{n} \quad \text{where } n = \frac{\pi}{6}$$

$$= 2\pi \times \frac{6}{\pi}$$

$$= 12 \text{ hours}$$

$\therefore$ The period of h is 12 hours.

b. Find h at low tide

$\Rightarrow h$ will be a minimum when

$$\sin\left(\frac{\pi}{6}t\right) = -1$$

$$\therefore\ h_{\text{min}} = 1 + 0.7(-1)$$

$$= 0.3 \text{ metres}$$

IMPORTANT: Using $\sin x = -1$ for a minimum here is very effective and time efficient. This property of trig functions is ***often very useful*** in harder questions.

Since $\sin x = -1$ when $x = \frac{3\pi}{2}$

$$\frac{\pi}{6}t = \frac{3\pi}{2}$$

$$t = \frac{3\pi}{2} \times \frac{6}{\pi}$$

$$= 9 \text{ hours}$$

$\therefore$ Low tide occurs at 2pm (5 am + 9 hours)

c. Find t when $h \geq 1.35$

$$1 + 0.7\sin\left(\frac{\pi}{6}t\right) \geq 1.35$$

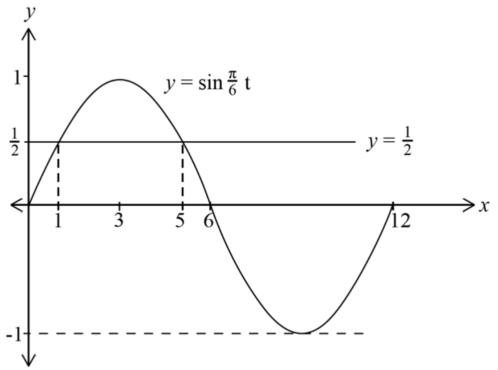
$$0.7\sin\left(\frac{\pi}{6}t\right) \geq 0.35$$

$$\sin\left(\frac{\pi}{6}t\right) \geq \frac{1}{2}$$

$\sin\left(\frac{\pi}{6}t\right) = \frac{1}{2}$ when

$$\frac{\pi}{6}t = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \text{ etc} \ldots$$

$$t = 1,\ 5 \qquad (0 \leq t \leq 12)$$



From the graph,

$$\sin\left(\frac{\pi}{6}t\right) \geq \frac{1}{2} \quad \text{when} \quad 1 \leq t \leq 5$$

$\therefore$ Ship can enter the harbour between 6 am and 10 am.

21. Trigonometry, 2ADV T2 2025 HSC 31

$$\cos(px) = \frac{1}{2} \quad \Rightarrow \quad px = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}$$

♦♦♦ Mean mark 27%.

Since there are 2 solutions in range $0 \leq x < 2\pi$:

$$px = \frac{5\pi}{3} \Rightarrow x = \frac{5\pi}{3p}$$

$$\frac{5\pi}{3p} \leqslant 2\pi$$

$$3p \geqslant \frac{5}{2}$$

$$p \geqslant \frac{5}{6}$$

Since $px = \dfrac{7\pi}{3}$ cannot be a solution in the range $0 \leq x \leq 2\pi$:

$$px = \frac{7\pi}{3} \Rightarrow x = \frac{7\pi}{3p}$$

$$\frac{7\pi}{3p} > 2\pi$$

$$p < \frac{7}{6}$$

$$\therefore \frac{5}{6} \leqslant p < \frac{7}{6}$$