MATHEMATICS
EXTENSION 1

SmarterEd Maths & Sciences

COMPREHENSIVE HSC REVISION SERIES 2025

Thank you for subscribing to SmarterMaths Teacher Edition (Silver) in 2025.

Key features of the Extension 1 “2025 HSC Comprehensive Revision Series” include:

- ~18 hours of cherry-picked HSC revision questions by topic
- Targeted at motivated students aiming for a Band 5 or 6 result
- Weighting toward more difficult examples
- Mark allocations given to each topic generally reflect its historical (new syllabus) HSC exam allocation.
- ***Attempt, carefully review and annotate*** this revision set in Term 3
- This question set provides the foundation of a concise and high-quality revision resource for the run into the HSC exam.
- This resource should be used to complement (not replace) the critical final stretch preparation for every student – completing full practice papers in exam conditions.

Our analysis on each topic, the common question types, past areas of difficulty and recent HSC trends all combine to create this revision set that ensures students cover a wide cross-section of the key areas.

IMPORTANT: If students have been exposed to questions in these worksheets during the year, we say great. Many top performing students attest to the benefits of doing quality questions 2-3 times before the HSC. This type of revision set is aimed at creating confidence and *speed through the exam*, with cherry-picked questions that cover all important elements of revision while avoiding low percentage rabbit hole excursions.

HSC Final Study: C3 Applications of Calculus (1 of 2)

(estimated ~17.1% of exam)

Key Areas addressed by this worksheet

Further Area and Solids of Revolution (7.1%)

- *Further Area and Solids of Revolution* has exceeded expectations by being examined in the longer answer section of every new syllabus exam to date, including twice in three of those years.
- *Solids of Revolution* are the dominant question type, which is reflected in the revision set. x-axis and y-axis rotations are both reviewed, with the latter historically causing more problems.
- This area is typically examined at the band 4 difficulty although this was ratcheted up by *2023 Q12e* which is a must review question.
- Question choice includes recent *Ext1* examples and others from the 2013-14 *Ext1* papers. The rest of the questions are chosen from past Advanced papers (identified by *EXT1** in their title).
- *Further Area* has now been examined in 3 of the last 4 years and deserves revision attention.

EXTENSION 1 2025

HSC Revision Series

Calculus

C3 Applications of Calculus (1 of 2)

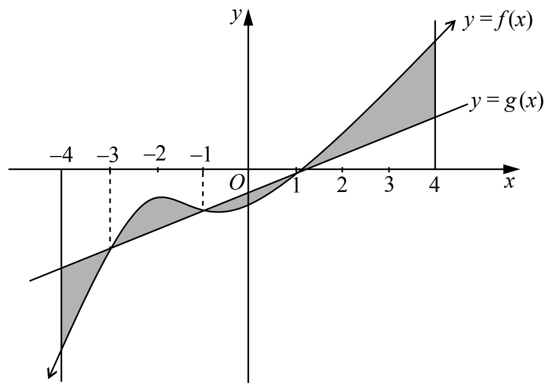
Further Area and Solids of Revolution (Y12)

Exam Equivalent Time: 75 minutes *(based on allocation of 1.5 minutes per mark)*

Questions

1. Calculus, EXT1 C3 2024 HSC 2 MC

Consider the functions $y=f(x)$ and $y=g(x)$, and the regions shaded in the diagram below.



Which of the following gives the total area of the shaded regions?

A. $\int_{-4}^{4} f(x)-g(x)\,dx$

B. $\left|\int_{-4}^{4} f(x)-g(x)\,dx\right|$

C. $\int_{-4}^{-3} f(x)-g(x)\,dx+\int_{-3}^{-1} f(x)-g(x)\,dx+\int_{-1}^{1} f(x)-g(x)\,dx+\int_{1}^{4} f(x)-g(x)\,dx$

D. $-\int_{-4}^{-3} f(x)-g(x)\,dx+\int_{-3}^{-1} f(x)-g(x)\,dx-\int_{-1}^{1} f(x)-g(x)\,dx+\int_{1}^{4} f(x)-g(x)\,dx$

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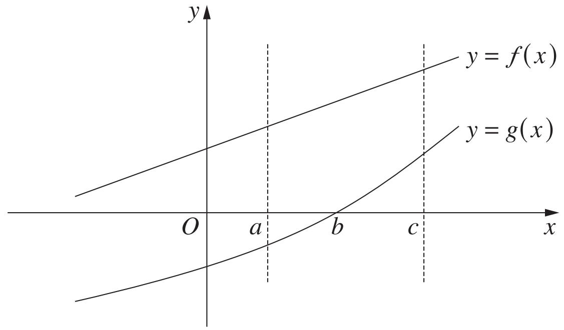
2. Calculus, EXT1 C3 2023 HSC 4 MC

The diagram shows the graphs of the functions $f(x)$ and $g(x)$.



It is known that

$$\int_a^c f(x)dx = 10$$
$$\int_a^b g(x)dx = -2$$
$$\int_b^c g(x)dx = 3.$$

What is the area between the curves $y=f(x)$ and $y=g(x)$ between $x=a$ and $x=c$?

A. 5

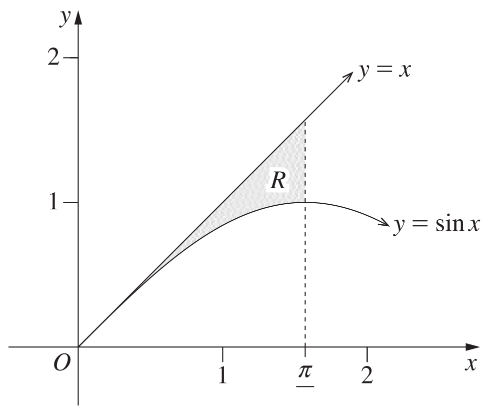
B. 7

C. 9

D. 11

3. Calculus, EXT1 C3 2024 HSC 11g

The region, R, is bounded by the curves $y = \sin x$, $y = x$ and the line $x = \frac{\pi}{2}$ as shown in the diagram.



Find the area of the region R. ***(3 marks)***

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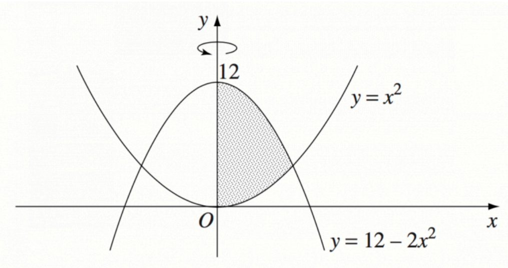
4. Calculus, EXT1* C3 2005 HSC 6c



The graphs of the curves $y = x^2$ and $y = 12 - 2x^2$ are shown in the diagram.

i. Find the points of intersection of the two curves. ***(1 mark)***

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ii. The shaded region between the curves and the y-axis is rotated about the y-axis. By splitting the shaded region into two parts, or otherwise, find the volume of the solid formed. ***(3 marks)***

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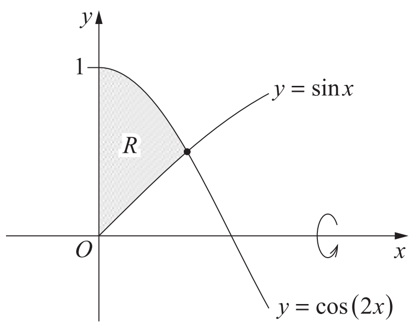
5. Calculus, EXT1 C3 2020 HSC 13b

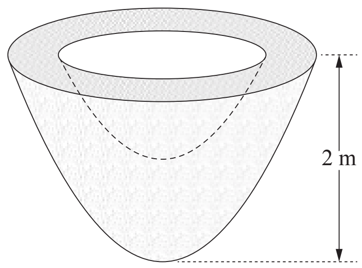
The region R is bounded by the y-axis, the graph of $y = \cos(2x)$ and the graph of $y = \sin x$, as shown in the diagram.



Find the volume of the solid of revolution formed when the region R is rotated about the x-axis. *(4 marks)*

6. Calculus, EXT1 C3 2021 HSC 13a

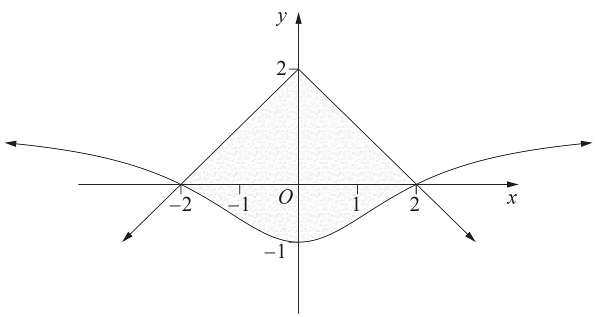
A 2-metre-high sculpture is to be made out of concrete. The sculpture is formed by rotating the region between $y = x^2$, $y = x^2 + 1$ and $y = 2$ around the y-axis.



Find the volume of concrete needed to make the sculpture. *(3 marks)*

7. Calculus, EXT1 C3 2021 HSC 13c

The region enclosed by $y = 2 - |x|$ and $y = 1 - \dfrac{8}{4 + x^2}$ is shaded in the diagram.



Find the exact value of the area of the shaded region. ***(3 marks)***

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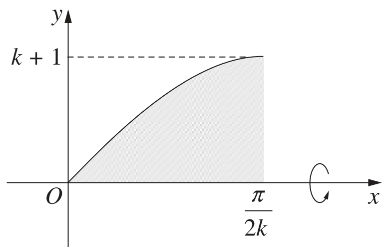
8. Calculus, EXT1 C3 2022 HSC 13b

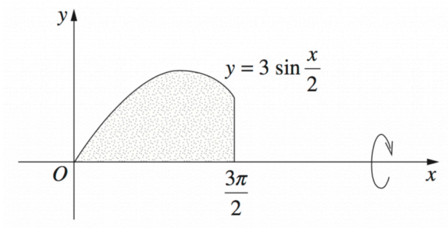
A solid of revolution is to be found by rotating the region bounded by the x-axis and the curve $y = (k + 1)\sin(kx)$, where $k > 0$, between $x = 0$ and $x = \dfrac{\pi}{2k}$ about the x-axis.



Find the value of k for which the volume is π^2. ***(3 marks)***

9. Calculus, EXT1 C3 2013 HSC 12b

The region bounded by the graph $y = 3\sin\frac{x}{2}$ and the x-axis between $x = 0$ and $x = \frac{3\pi}{2}$ is rotated about the x-axis to form a solid.



Find the exact volume of the solid. *(3 marks)*

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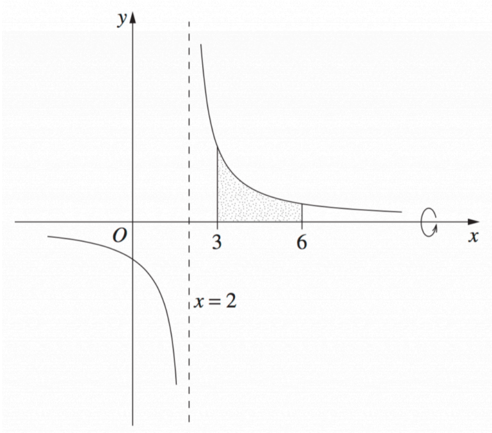
10. Calculus, EXT1* C3 2008 HSC 6c

The graph of $y = \frac{5}{x-2}$ is shown below.



The shaded region in the diagram is bounded by the curve $y = \frac{5}{x-2}$, the x-axis and the lines $x = 3$ and $x = 6$.

Find the volume of the solid of revolution formed when the shaded region is rotated about the x-axis. *(3 marks)*

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11. Calculus, EXT1 C3 2005 HSC 5a

Find the exact value of the volume of the solid of revolution formed when the region bounded by the curve $y = \sin 2x$, the x-axis and the line $x = \dfrac{\pi}{8}$ is rotated about the x-axis. *(3 marks)*

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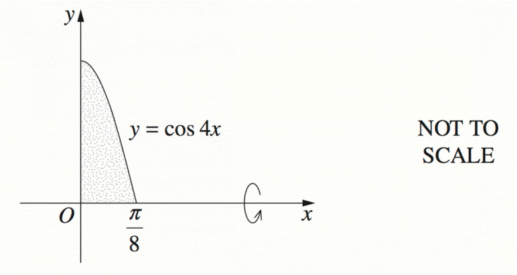
12. Calculus, EXT1 C3 2014 HSC 12b

The region bounded by $y = \cos 4x$ and the x-axis, between $x = 0$ and $x = \dfrac{\pi}{8}$, is rotated about the x-axis to form a solid.



Find the volume of the solid. *(3 marks)*

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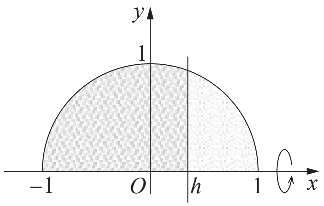
13. Calculus, EXT1 C3 SM-Bank 1

The region enclosed by the semicircle $y = \sqrt{1 - x^2}$ and the x-axis is to be divided into two pieces by the line $x = h$, when $0 \le h < 1$.



The two pieces are rotated about the x-axis to form solids of revolution. The value of h is chosen so that the volumes of the solids are in the ratio $2:1$.

Show that h satisfies the equation $3h^3 - 9h + 2 = 0$. *(3 marks)*

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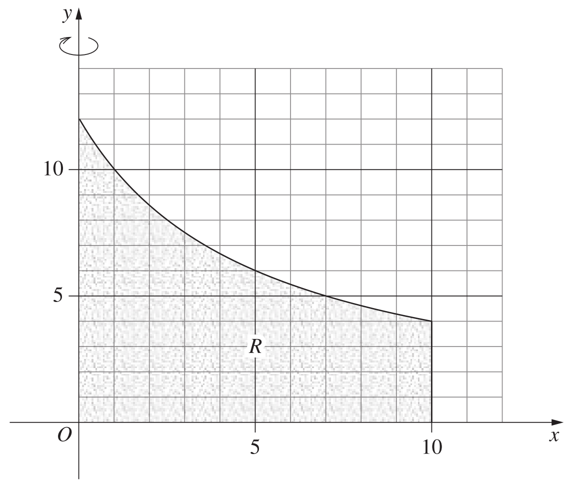
14. Calculus, EXT1 C3 2023 HSC 12e

The region, R, bounded by the hyperbola $y = \dfrac{60}{x+5}$, the line $x = 10$ and the coordinate axes is shown.



Find the volume of the solid of revolution formed when the region R is rotated about the y-axis. Leave your answer in exact form. *(4 marks)*

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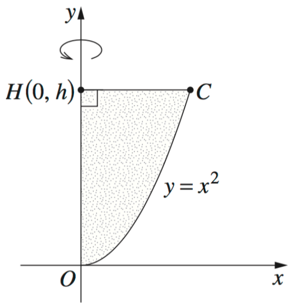
15. Calculus, EXT1* C3 2011 HSC 8b

The diagram shows the region enclosed by the parabola $y = x^2$, the y-axis and the line $y = h$, where $h > 0$. This region is rotated about the y-axis to form a solid called a paraboloid. The point C is the intersection of $y = x^2$ and $y = h$.

The point H has coordinates $(0, h)$.



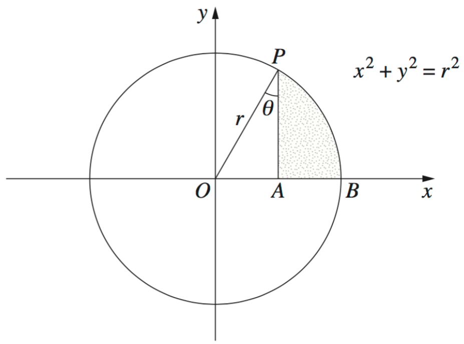
i. Find the exact volume of the paraboloid in terms of h. *(2 marks)*

ii. A cylinder has radius HC and height h.

What is the ratio of the volume of the paraboloid to the volume of the cylinder? *(1 mark)*

16. Calculus, EXT1* C3 2010 HSC 10b

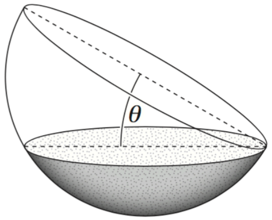
The circle $x^2 + y^2 = r^2$ has radius r and centre O. The circle meets the positive x-axis at B. The point A is on the interval OB. A vertical line through A meets the circle at P. Let $\theta = \angle OPA$.



i. The shaded region bounded by the arc PB and the intervals AB and AP is rotated about the x-axis. Show that the volume, V, formed is given by

$V = \frac{\pi r^3}{3}\left(2 - 3\sin\theta + \sin^3\theta\right)$ *(3 marks)*

ii. A container is in the shape of a hemisphere of radius r metres. The container is initially horizontal and full of water. The container is then tilted at an angle of θ to the horizontal so that some water spills out.



(1) Find θ so that the depth of water remaining is one half of the original depth. ***(1 mark)***

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(2) What fraction of the original volume is left in the container? ***(2 marks)***

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Worked Solutions

1. Calculus, EXT1 C3 2024 HSC 2 MC

Intervals where $f(x) > g(x) \;\Rightarrow$ Positive area values

Intervals where $g(x) > f(x) \;\Rightarrow$ Negative area values

$\Rightarrow D$

2. Calculus, EXT1 C3 2023 HSC 4 MC

$$A = \int_a^c f(x)\,dx - \int_b^c g(x)\,dx + \left|\int_a^b g(x)\,dx\right|$$

$$= 10 - 3 + |-2|$$

$$= 9$$

$\Rightarrow C$

3. Calculus, EXT1 C3 2024 HSC 11g

$$R = \int_0^{\frac{\pi}{2}} x - \sin x\,dx$$

$$= \left[\frac{x^2}{2} + \cos x\right]_0^{\frac{\pi}{2}}$$

$$= \left[\left(\frac{\pi^2}{8} + \cos\frac{\pi}{2}\right) - (0 + \cos 0)\right]$$

$$= \frac{\pi^2}{8} - 1 \text{ u}^2$$

4. Calculus, EXT1* C3 2005 HSC 6c

i. $y = x^2 \quad \ldots (1)$

$y = 12 - 2x^2 \ldots (2)$

Substitute $y = x^2$ into (2)

$$x^2 = 12 - 2x^2$$
$$3x^2 - 12 = 0$$
$$3(x^2 - 4) = 0$$
$$x = \pm 2$$

When $x = 2, \quad y = 4$

When $x = -2, \ y = 4$

$\therefore$ Intersection at $(2, 4), (-2, 4)$

ii. In $(1), \ x^2 = y$

In $(2), \quad y = 12 - 2x^2$

$$2x^2 = 12 - y$$
$$x^2 = \frac{12 - y}{2}$$
$$= 6 - \frac{1}{2}y$$

$\therefore$ Volume

$$= \pi \int_0^4 y \, dy + \pi \int_4^{12} 6 - \frac{1}{2}y \, dy$$
$$= \pi \left[\frac{y^2}{2}\right]_0^4 + \pi \left[6y - \frac{y^2}{4}\right]_4^{12}$$
$$= \pi \left[\frac{16}{2} - 0\right] + \pi \left[\left(6 \times 12 - \frac{12^2}{4}\right) - \left(6 \times 4 - \frac{4^2}{4}\right)\right]$$
$$= 8\pi + \pi[36 - 20]$$
$$= 8\pi + 16\pi$$
$$= 24\pi \ \text{u}^3$$

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5. Calculus, EXT1 C3 2020 HSC 13b

Mean mark 53%.

Find intersection:

$$\sin x = \cos 2x$$
$$\sin x = 1 - 2\sin^2 x$$
$$2\sin^2 x + \sin x - 1 = 0$$
$$(2\sin x - 1)(\sin x + 1) = 0$$

$\sin x = \dfrac{1}{2} \quad$ or $\quad \sin x = -1$

$x = \dfrac{\pi}{6} \qquad\qquad x = \dfrac{3\pi}{2}$

$$V = \pi \int_0^{\frac{\pi}{6}} (\cos 2x)^2 \, dx - \pi \int_0^{\frac{\pi}{6}} (\sin x)^2 \, dx$$
$$= \pi \int_0^{\frac{\pi}{6}} \cos^2 2x - \sin^2 x \, dx$$
$$= \pi \int_0^{\frac{\pi}{6}} \frac{1}{2}(1 + \cos 4x) - \frac{1}{2}(1 - \cos 2x) \, dx$$
$$= \frac{\pi}{2} \int_0^{\frac{\pi}{6}} \cos 4x + \cos 2x \, dx$$
$$= \frac{\pi}{2}\left[\frac{1}{4}\sin 4x + \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{6}}$$
$$= \frac{\pi}{8}\left[\sin \frac{2\pi}{3} + 2\sin \frac{\pi}{3}\right]$$
$$= \frac{\pi}{8}\left(\frac{\sqrt{3}}{2} + 2 \times \frac{\sqrt{3}}{2}\right)$$
$$= \frac{3\sqrt{3}\pi}{16} \ \text{u}^3$$

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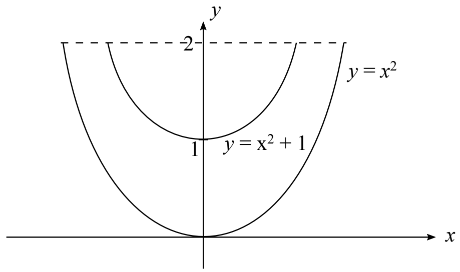
6. Calculus, EXT1 C3 2021 HSC 13a



$$V = \pi\int_0^2 y\ dy - \pi\int_1^2 y - 1\ dy$$
$$= \pi\left[\frac{y^2}{2}\right]_0^2 - \pi\left[\frac{y^2}{2} - y\right]_1^2$$
$$= \pi(2-0) - \pi\left[(2-2) - \left(\frac{1}{2} - 1\right)\right]$$
$$= 2\pi - \pi\left(\frac{1}{2}\right)$$
$$= \frac{3\pi}{2}\ \text{u}^3$$

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7. Calculus, EXT1 C3 2021 HSC 13c

$$A = \text{Area of } \Delta + 2\left|\int_0^2 1 - \frac{8}{4+x^2}\ dx\right|$$
$$= \frac{1}{2} \times 4 \times 2 + 2\left|\left[x - 4\tan^{-1}\ \frac{x}{2}\right]_0^2\right|$$
$$= 4 + 2\left|(2 - 4\tan^{-1}1) - 0\right|$$
$$= 4 + 2\left|2 - \frac{4\pi}{4}\right|$$
$$= 4 + 2(\pi - 2)$$
$$= 2\pi\ \text{u}^2$$

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8. Calculus, EXT1 C3 2022 HSC 13b

$$y = (k+1)\sin(kx)$$
$$V = \pi\int_0^{\frac{\pi}{2k}} (k+1)^2\sin^2(kx)\ dx$$
$$= \pi(k+1)^2\int_0^{\frac{\pi}{2k}} \frac{1}{2}[1 - \cos(2kx)]\ dx$$
$$= \left(\frac{\pi}{2}\right)(k+1)^2\left[x - \frac{\sin(2kx)}{2k}\right]_0^{\frac{\pi}{2k}}$$
$$= \left(\frac{\pi}{2}\right)(k+1)^2\left[\left(\frac{\pi}{2k} - \frac{\sin(\pi)}{2k}\right) - \left(0 - \frac{\sin 0}{2k}\right)\right]$$
$$= \left(\frac{\pi}{2}\right)(k+1)^2\left(\frac{\pi}{2k}\right)$$
$$= \frac{\pi^2}{4k}(k+1)^2$$

Given $V = \pi^2$:

$$\frac{\pi^2}{4k}(k+1)^2 = \pi^2$$
$$(k+1)^2 = 4k$$
$$k^2 + 2k + 1 = 4k$$
$$k^2 - 2k + 1 = 0$$
$$(k-1)^2 = 0$$

$$\therefore k = 1$$

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9. Calculus, EXT1 C3 2013 HSC 12b

$$y = 3\sin\ \frac{x}{2}$$
$$y^2 = 9\sin^2\ \frac{x}{2}$$

$$\text{Using:}\ \ \sin^2 x = \frac{1}{2}(1-\cos 2x)$$

$$\begin{aligned}\therefore V &= \pi\int_0^{\frac{3\pi}{2}} 9\sin^2\ \frac{x}{2}\ dx \\ &= \frac{9\pi}{2}\int_0^{\frac{3\pi}{2}} (1-\cos x)\ dx \\ &= \frac{9\pi}{2}[x-\sin x]_0^{\frac{3\pi}{2}} \\ &= \frac{9\pi}{2}\left[\left(\frac{3\pi}{2}-\sin\ \frac{3\pi}{2}\right)-0\right] \\ &= \frac{9\pi}{2}\left(\frac{3\pi}{2}+1\right)\ \text{u}^3\end{aligned}$$

10. Calculus, EXT1* C3 2008 HSC 6c

$$y = \frac{5}{x-2}$$

$$\begin{aligned}V &= \pi\int_3^6 y^2\ dx \\ &= \pi\int_3^6 \left(\frac{5}{x-2}\right)^2\ dx \\ &= 25\pi\int_3^6 \frac{1}{(x-2)^2}\ dx \\ &= 25\pi\left[\frac{-1}{x-2}\right]_3^6 \\ &= 25\pi\left[-\frac{1}{4}-(-1)\right] \\ &= 25\pi\left[\frac{3}{4}\right] \\ &= \frac{75\pi}{4}\ \text{u}^3\end{aligned}$$

11. Calculus, EXT1 C3 2005 HSC 5a

$$y = \sin 2x$$
$$y^2 = \sin^2 2x$$

$$\text{Using:}\ \ \sin^2 x = \frac{1}{2}(1-\cos 2x)$$

COMMENT: Michael Wells (1st in state Ext2) would derive this formula in his working from the $\sin^2 x + \cos^2 = 1$ identity in ~5 seconds *every time* he used it in an exam.

$$\begin{aligned}\therefore V &= \pi\int_0^{\frac{\pi}{8}} y^2\ dx \\ &= \pi\int_0^{\frac{\pi}{8}} \sin^2 2x\ dx \\ &= \frac{\pi}{2}\int_0^{\frac{\pi}{8}} 1-\cos\ 4x\ dx \\ &= \frac{\pi}{2}\left[x-\frac{1}{4}\sin\ 4x\right]_0^{\frac{\pi}{8}} \\ &= \frac{\pi}{2}\left[\left(\frac{\pi}{8}-\frac{1}{4}\sin\ \frac{\pi}{2}\right)-0\right] \\ &= \frac{\pi}{2}\left(\frac{\pi}{8}-\frac{1}{4}\right) \\ &= \frac{\pi}{2}\left(\frac{\pi-2}{8}\right) \\ &= \frac{\pi}{16}(\pi-2)\quad \text{u}^3\end{aligned}$$

12. Calculus, EXT1 C3 2014 HSC 12b

$$\begin{aligned} V &= \pi \int_0^{\frac{\pi}{8}} y^2 \ dx \\ &= \pi \int_0^{\frac{\pi}{8}} \cos^2 4x \ dx \\ &= \pi \int_0^{\frac{\pi}{8}} \frac{1}{2}(\cos 8x + 1) \ dx \\ &= \frac{\pi}{2}\left[\frac{1}{8}\sin 8x + x\right]_0^{\frac{\pi}{8}} \\ &= \frac{\pi}{2}\left[\left(\frac{1}{8}\sin\pi + \frac{\pi}{8}\right) - 0\right] \\ &= \frac{\pi^2}{16} \ \text{u}^3 \end{aligned}$$

COMMENT: The identities $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$ are tested every year – *know them.*

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13. Calculus, EXT1 C3 SM-Bank 1

(i) Volume of smaller solid

$$\begin{aligned} &= \pi \int_h^1 \left(\sqrt{1 - x^2}\right)^2 \ dx \\ &= \pi \int_h^1 1 - x^2 \ dx \\ &= \pi \left[x - \frac{x^3}{3}\right]_h^1 \\ &= \pi\left[\left(1 - \frac{1}{3}\right) - \left(h - \frac{h^3}{3}\right)\right] \\ &= \pi\left(\frac{2}{3} - h + \frac{h^3}{3}\right) \end{aligned}$$

Since smaller solid is $\frac{1}{3}$ volume of sphere,

$$\begin{aligned} \pi\left(\frac{2}{3} - h + \frac{h^3}{3}\right) &= \frac{1}{3} \times \frac{4}{3} \cdot \pi \cdot 1^3 \\ \frac{h^3}{3} - h + \frac{2}{3} &= \frac{4}{9} \\ 3h^3 - 9h + 6 &= 4 \\ \therefore 3h^3 - 9h + 2 &= 0 \quad \ldots \text{as required} \end{aligned}$$

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14. Calculus, EXT1 C3 2023 HSC 12e

$$y = \frac{60}{x + 5} \quad \Rightarrow \quad x = \frac{60}{y} - 5$$

♦ Mean mark 49%.

$$\begin{aligned} V &= \pi \int_4^{12} x^2 \ dy + \pi r^2 h \\ &= \pi \int_4^{12} \left(\frac{60}{y} - 5\right)^2 dy + \pi \times 10^2 \times 4 \\ &= 25\pi \int_4^{12} \left(\frac{12}{y} - 1\right)^2 dy + 400\pi \\ &= 25\pi \int_4^{12} \left(\frac{144}{y^2} - \frac{24}{y} + 1\right) dy + 400\pi \\ &= 25\pi\left[\frac{-144}{y} - 24\ln y + y\right]_4^{12} + 400\pi \\ &= 25\pi\Big[(-12 - 24\ln 12 + 12) - (-36 - 24\ln 4 + 4)\Big] + 400\pi \\ &= 25\pi(24\ln 4 - 24\ln 12 + 32) + 400\pi \\ &= 600\pi \ln\left(3^{-1}\right) + 800\pi + 400\pi \\ &= 1200\pi - 600\pi \ln 3 \ \ \text{u}^3 \end{aligned}$$

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15. Calculus, EXT1* C3 2011 HSC 8b

IMPORTANT: Most common errors: *1-use the correct axis, and 2-check the limits!*

i. $V = \pi \int_0^h x^2 \ dy$

$= \pi \int_0^h y \ dy$

$= \pi \left[\frac{1}{2} y^2 \right]_0^h$

$= \pi \left(\frac{1}{2} h^2 \right)$

$= \frac{\pi h^2}{2}$ u³

$\therefore$ The volume of the paraboloid is $\frac{\pi h^2}{2}$ u³

ii. Radius of cylinder $(r) = HC$

♦♦ Mean mark of 24%.

Find x-coordinate of C:

When $y = h$

$\Rightarrow x^2 = h$

$x = \sqrt{h}$

$\therefore r = \sqrt{h}$

Volume of cylinder $= \pi r^2 h$

$= \pi \left(\sqrt{h} \right)^2 h$

$= \pi h^2$

$\therefore$ Volume of paraboloid : volume of cylinder

$= \frac{\pi h^2}{2} : \pi h^2$

$= 1:2$

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16. Calculus, EXT1* C3 2010 HSC 10b

i. Show that $V = \frac{\pi r^3}{3} (2 - 3\sin\theta + \sin^3\theta)$

♦♦♦ Mean mark (i) 16%.
MARKER'S COMMENT: A common error was to integrate r^2 to $\frac{1}{3}r^3$ instead of $r^2 x$ (note that r is a constant).

$\sin\theta = \frac{OA}{r}$

$\therefore \ OA = r\sin\theta$

$\Rightarrow A = (r\sin\theta, 0), \quad B = (r, 0)$

$\therefore V = \pi \int_{r\sin\theta}^{r} y^2 \ dx$

$= \pi \int_{r\sin\theta}^{r} (r^2 - x^2) \ dx \quad (\text{using } x^2 + y^2 = r^2)$

$= \pi \left[r^2 x - \frac{x^3}{3} \right]_{r\sin\theta}^{r}$

$= \pi \left[\left(r^3 - \frac{r^3}{3} \right) - \left(r^3 \sin\theta - \frac{r^3 \sin^3\theta}{3} \right) \right]$

$= \pi \left(\frac{2r^3}{3} - r^3 \sin\theta + \frac{r^3 \sin^3\theta}{3} \right)$

$= \frac{\pi r^3}{3} (2 - 3\sin\theta + \sin^3\theta) \quad \ldots$ as required

ii. (1) Depth of water remaining $= \frac{1}{2} \times$ original depth:

♦♦♦ Part (ii) mean marks 3% and 2% for (ii)(1) and (ii)(2) respectively.

$r - r\sin\theta = \frac{1}{2} r$

$r(1 - \sin\theta) = \frac{1}{2} r$

$1 - \sin\theta = \frac{1}{2}$

$\sin\theta = \frac{1}{2}$

$\therefore \ \theta = \frac{\pi}{6}$ radians

ii. (2) Original Volume $= \frac{1}{2} \times \frac{4}{3} \pi r^3$

$= \frac{2}{3} \pi r^3$

MARKER'S COMMENT: Previous parts of a question should always be at the front and centre of a student's mind and direct their strategy.

New Volume $= \frac{\pi r^3}{3} \left[2 - 3\sin\left(\frac{\pi}{6}\right) + \sin^3\left(\frac{\pi}{6}\right) \right]$

$$= \frac{\pi r^3}{3}\left[2 - \left(3 \times \frac{1}{2}\right) + \left(\frac{1}{2}\right)^3\right]$$

$$= \frac{\pi r^3}{3}\left[2 - \frac{3}{2} + \frac{1}{8}\right]$$

$$= \frac{\pi r^3}{3}\left(\frac{5}{8}\right)$$

$$= \frac{5\pi r^3}{24}$$

$\therefore$ Fraction of original volume left

$$= \frac{\frac{5\pi r^3}{24}}{\frac{2}{3}\pi r^3}$$

$$= \frac{5}{24} \times \frac{3}{2}$$

$$= \frac{5}{16}$$