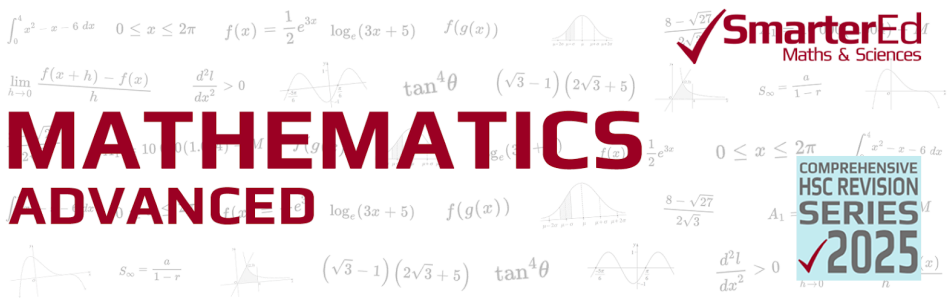


Thank you for subscribing to SmarterMaths Teacher Edition (Silver) in 2025.

Key features of the Advanced "2025 HSC Comprehensive Revision Series" for include:

- ~22 hours of cherry picked HSC revision questions by topic.
- Targeted at motivated students aiming for a Band 5 or 6 result
- Weighting toward more difficult examples.
- Mark allocations given to each topic generally reflect its historical (new syllabus) HSC exam allocation.
- ***Attempt, carefully review and annotate*** this revision set in Term 3.
- This question set provides the foundation of a concise and high quality revision resource for the run into the HSC exam.
- This resource should be used to complement (not replace) the critical final stretch preparation for every student - timed full exam practice papers.

Our analysis on each topic, the common question types, past areas of difficulty and recent HSC trends all combine to create this revision set that ensures students cover a wide cross-section of the key areas.

IMPORTANT: If students have been exposed to questions in these worksheets during the year, we say great. Many top performing students attest to the benefits of doing quality questions 2-3 times before the HSC. This type of revision set is aimed at creating confidence and *speed through the exam*, with cherry picked questions that cover all important elements of revision while avoiding low percentage rabbit hole excursions.

HSC Final Study: TRIGONOMETRY (estimated ~7.2% of exam)

Key Areas addressed by this worksheet

T2 Trig Functions and Identities (1.2%)

- A small contributor that remains important due to the critical gateway knowledge it covers for many later topics.
- *Exact Trig Ratios* most recently attracted a 3-mark question in 2023 after a long absence. Note the use of "dedicated" here as this concept is tested multiple times every year as part of applied topics covered later in the revision series.
- The most common question type requires students to solve a simple trig equation in exact radian form within a specified range. Numerous examples look at this, including the harder *2008 HSC 6a* which caused problems.
- Questions giving an exact trig ratio that require students to use Pythagoras and find exact *sec, cosec* and *cot* ratios is covered (similar to NESA Topic Guidance exemplar questions).
- *Trig Identities and Harder Equations*: examined most recently in 2020-21 and absent since.
- This topic can be examined in a broad array of questions that are often challenging and this revision set features multiple harder examples.

T3 Trig Functions and Graphs (6.0%)

- *Trig Graphs:* examined in each new syllabus exam between 2020-22 and notably absent in the last two years.
- *Trig Graphs* will involve transformations and require students to sketch or recognize these graphs. Multiple examples cover this area.
- Amplitude and period of various trig functions is well covered (including the period of *tan* functions which has caused problems in the past).
- We also revise examples that reflect NESA sample exam questions in this topic area.
- *Trig Applications:* examined with significant allocations of 6-7 marks each year in the period 2022-24 (note the 2021 question was a large cross-topic example incorporating integration which will be covered in a later topic).
- With high mark allocations and low mean marks, this topic area represents critical revision which is reflected in the revision set.

> *"When the mind grows weary from trigonometry, step outside and look up and let clarity come not by force, but by space."*
>
> *~ Marcus Aurelius*

ADVANCED 2025

HSC Revision Series

Trigonometry

T2 Trig Functions and Identities (Y11)

T3 Trig Functions and Graphs (Y12)

Exam Equivalent Time: 90 minutes *(based on allocation of 1.5 minutes per mark)*

Questions

1. Trigonometry, 2ADV T3 2020 HSC 6 MC

Which interval gives the range of the function $y = 5 + 2\cos 3x$?

A. $[2, 8]$

B. $[3, 7]$

C. $[4, 6]$

D. $[5, 9]$

---

2. Trigonometry, 2ADV T2 2012 HSC 6 MC

What are the solutions of $\sqrt{3}\tan x = -1$ for $0 \leq x \leq 2\pi$?

A. $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$

B. $\frac{2\pi}{3}$ and $\frac{5\pi}{3}$

C. $\frac{5\pi}{6}$ and $\frac{7\pi}{6}$

D. $\frac{5\pi}{6}$ and $\frac{11\pi}{6}$

---

3. Trigonometry, 2ADV T2 2017 HSC 7 MC

Which expression is equivalent to $\tan\theta + \cot\theta$?

A. $\operatorname{cosec}\ \theta + \sec\theta$

B. $\sec\theta\ \operatorname{cosec}\ \theta$

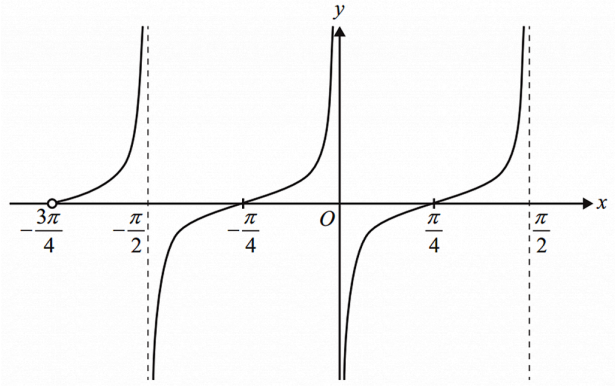
C. 2

D. 1

---

4. Trigonometry, 2ADV T3 SM-Bank 4 MC

A section of the graph of $f(x)$ is shown below.



The equation of $f(x)$ could be

A. $f(x) = \tan(x)$

B. $f(x) = \tan\left(x - \frac{\pi}{4}\right)$

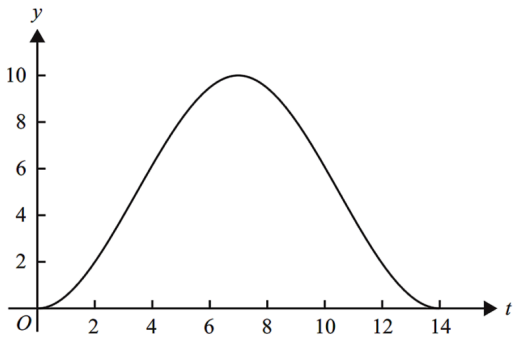
C. $f(x) = \tan\left(2\left(x - \frac{\pi}{4}\right)\right)$

D. $f(x) = \tan\left(2\left(x - \frac{\pi}{2}\right)\right)$

---

5. Trigonometry, 2ADV T3 SM-Bank 5 MC

The UV index, y, for a summer day in Newcastle East is illustrated in the graph below, where t is the number of hours after 6 am.



The graph is most likely to be the graph of

A. $y = 5 + 5\cos\left(\frac{\pi t}{7}\right)$

B. $y = 5 - 5\cos\left(\frac{\pi t}{7}\right)$

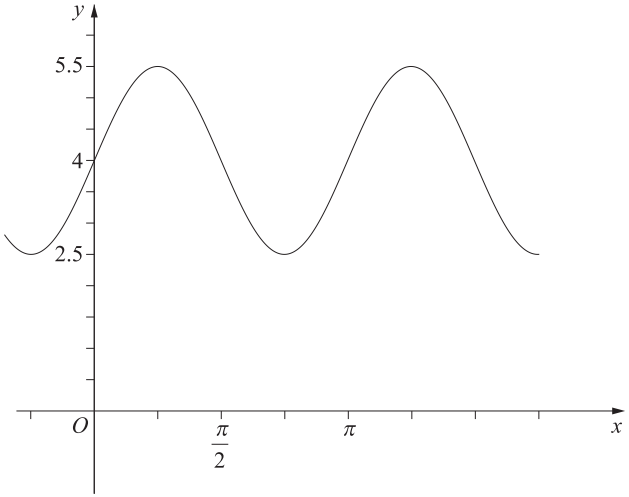
C. $y = 5 + 5\cos\left(\frac{\pi t}{14}\right)$

D. $y = 5 - 5\cos\left(\frac{\pi t}{14}\right)$

---

6. Trigonometry, 2ADV T3 2019 HSC 7 MC

The diagram shows part of the graph of $y = a\sin(bx) + 4$.



What are the values of a and b?

A. $a = 3 \quad b = \frac{1}{2}$

B. $a = 3 \quad b = 2$

C. $a = 1.5 \quad b = \frac{1}{2}$

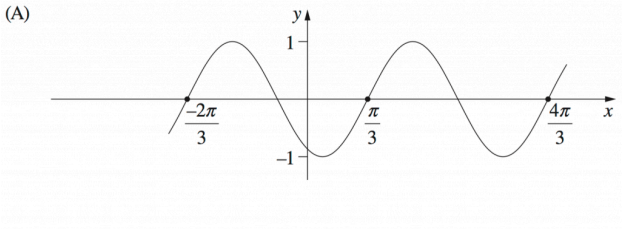
D. $a = 1.5 \quad b = 2$

---

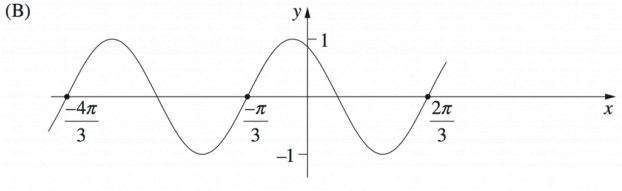
7. Trigonometry, 2ADV T3 2013 HSC 6 MC

Which diagram shows the graph $y = \sin\left(2x + \frac{\pi}{3}\right)$?

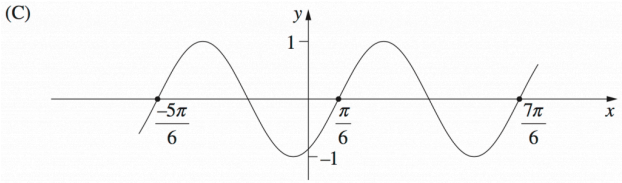
(A)



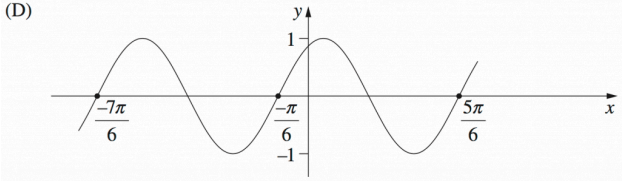
(B)



(C)



(D)



---

8. Trigonometry, 2ADV T3 2016 HSC 6 MC

What is the period of the function $f(x) = \tan(3x)$?

A. $\frac{\pi}{3}$

B. $\frac{2\pi}{3}$

C. 3π

D. 6π

---

9. Trigonometry, 2ADV T2 EQ-Bank 1 MC

Determine the number of values of θ in the range $0^\circ \leqslant \theta \leqslant 360^\circ$ that satisfy the equation

$$(\tan\theta - \sqrt{3})(\cos^2\theta - 1) = 0$$

A. 3

B. 4

C. 5

D. 6

---

10. Trigonometry, 2ADV T2 2016 HSC 8 MC

How many solutions does the equation $|\cos(2x)| = 1$ have for $0 \leq x \leq 2\pi$?

A. 1

B. 3

C. 4

D. 5

---

11. Trigonometry, 2ADV T2 EQ-Bank 5

Solve $\sin x - \cos x = 0 \quad -\pi \leqslant x \leqslant \pi$ *(2 marks)*

---

12. Trigonometry, 2ADV T2 EQ-Bank 7

Prove $\frac{1 + \cot\theta}{1 + \tan\theta} = \cot\theta$. *(3 marks)*

---

13. Trigonometry, 2ADV T2 SM-Bank 33

Given $\sec\theta = -\dfrac{37}{12}$ for $0 < \theta < \pi$,

find the exact value of $\operatorname{cosec}\ \theta$. *(2 marks)*

---

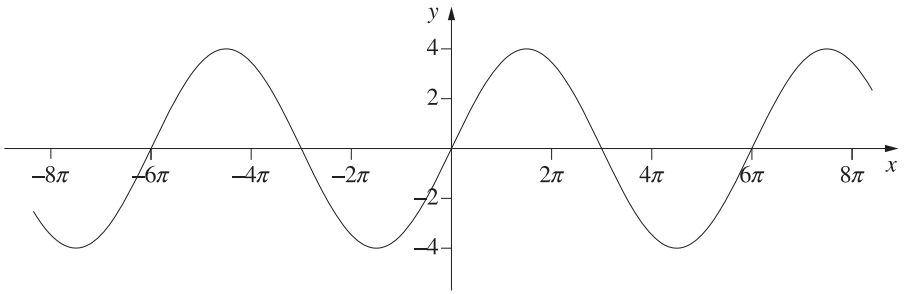
14. Trigonometry, 2ADV T3 2021 HSC 20

For what values of x, in the interval $0 \leq x \leq \dfrac{\pi}{4}$, does the line $y = 1$ intersect the graph of $y = 2\sin 4x$? *(2 marks)*

---

15. Trigonometry, 2ADV T3 2022 HSC 14

The graph of $y = k\sin(ax)$



What are the values of k and a? *(2 marks)*

---

16. Trigonometry, 2ADV T3 SM-Bank 12

State the range and period of the function

$h(x) = 4 + 3\cos\left(\dfrac{\pi x}{2}\right)$. *(2 marks)*

---

17. Trigonometry, 2ADV T2 2023 HSC 20

Find all the values of θ, where $0^\circ \leq \theta \leq 360^\circ$, such that

$\sin(\theta - 60^\circ) = -\dfrac{\sqrt{3}}{2}$ *(3 marks)*

---

18. Trigonometry, 2ADV T3 2018 HSC 15a

The length of daylight, $L(t)$, is defined as the number of hours from sunrise to sunset, and can be modelled by the equation

$$L(t) = 12 + 2\cos\left(\frac{2\pi t}{366}\right),$$

where t is the number of days after 21 December 2015, for $0 \leq t \leq 366$.

i. Find the length of daylight on 21 December 2015. *(1 mark)*

ii. What is the shortest length of daylight? *(1 mark)*

iii. What are the two values of t for which the length of daylight is 11? *(2 marks)*

---

19. Trigonometry, 2ADV T3 2013 HSC 13a

The population of a herd of wild horses is given by

$$P(t) = 400 + 50\cos\left(\frac{\pi}{6}t\right)$$

where t is time in months.

i. Find all times during the first 12 months when the population equals 375 horses. *(2 marks)*

ii. Sketch the graph of $P(t)$ for $0 \leq t \leq 12$. *(2 marks)*

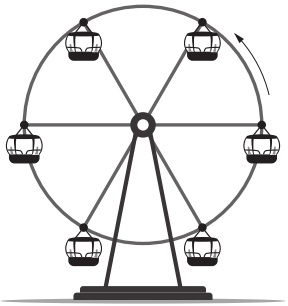
---

20. Trigonometry, 2ADV T3 2024 HSC 28

Anna is sitting in a carriage of a Ferris wheel which is revolving. The height, $A(t)$, in metres above the ground of the top of her carriage is given by

$$A(t) = c - k\cos\left(\frac{\pi t}{24}\right),$$

where t is the time in seconds after Anna's carriage first reaches the bottom of its revolution and c and k are constants.



NOT TO SCALE

The top of each carriage reaches a greatest height of 39 metres and a smallest height of 3 metres.

a. Find the value of c and k. ***(2 marks)***

b. How many seconds does it take for one complete revolution of the Ferris wheel? ***(1 mark)***

c. Billie is in another carriage. The height, $B(t)$, in metres above the ground of the top of her carriage is given by

$$B(t) = c - k\cos\left(\frac{\pi}{24}(t-6)\right),$$

where c and k are as found in part (a).

During each revolution, there are two occasions when Anna's and Billie's carriages are at the same heights. At what two heights does this occur? Give your answer correct to 2 decimal places. ***(4 marks)***

---

21. Trigonometry, 2ADV T2 2014 HSC 15a

Find all solutions of $2\sin^2 x + \cos x - 2 = 0$, where $0 \le x \le 2\pi$. ***(3 marks)***

---

22. Trigonometry, 2ADV T2 2008 HSC 6a

Solve $2\sin^2\left(\frac{x}{3}\right) = 1$ for $-\pi \le x \le \pi$. ***(3 marks)***

---

23. Trigonometry, 2ADV T2 2019 HSC 13a

Solve $2\sin x\cos x = \sin x$ for $0 \le x \le 2\pi$. ***(3 marks)***

---

24. Trigonometry, 2ADV T2 EQ-Bank 4

Prove $\dfrac{\operatorname{cosec}\theta + \sec\theta}{1 + \tan\theta} = \operatorname{cosec}\theta$. ***(3 marks)***

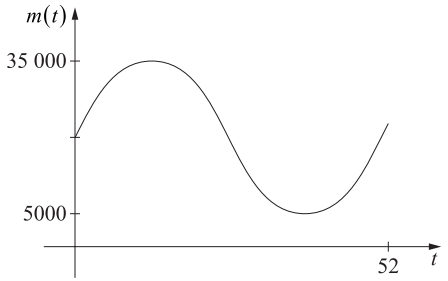
---

25. Trigonometry, 2ADV T3 2020 HSC 31

The population of mice on an isolated island can be modelled by the function.

$$m(t) = a\sin\left(\frac{\pi}{26}t\right) + b,$$

where t is the time in weeks and $0 \le t \le 52$. The population of mice reaches a maximum of 35 000 when $t = 13$ and a minimum of 5000 when $t = 39$. The graph of $m(t)$ is shown.



a. What are the values of a and b? ***(2 marks)***

b. On the same island, the population of cats can be modelled by the function

$$c(t) = -80\cos\left(\frac{\pi}{26}(t-10)\right) + 120$$

Consider the graph of $m(t)$ and the graph of $c(t)$.

Find the values of t, $0 \le t \le 52$, for which both populations are increasing. ***(3 marks)***

c. Find the rate of change of the mice population when the cat population reaches a maximum. ***(2 marks)***

Copyright © 2004-24 The State of New South Wales (Board of Studies, Teaching and Educational Standards NSW)

Worked Solutions

1. Trigonometry, 2ADV T3 2020 HSC 6 MC

$-1 \leq \cos 3x \leq 1$

$-2 \leq 2\cos 3x \leq 2$

$3 \leq 5 + 2\cos 3x \leq 7$

$\therefore$ Range $[3, 7]$

$\Rightarrow B$

---

2. Trigonometry, 2ADV T2 2012 HSC 6 MC

$\sqrt{3}\tan x = -1$

$\tan x = -\dfrac{1}{\sqrt{3}}$

When $\tan x = \dfrac{1}{\sqrt{3}}$, $x = \dfrac{\pi}{6}$

Since $\tan x$ is negative in $2^{\text{nd}}/4^{\text{th}}$ quadrant

$\therefore x = \pi - \dfrac{\pi}{6},\ 2\pi - \dfrac{\pi}{6},\ \ldots$

$= \dfrac{5\pi}{6},\ \dfrac{11\pi}{6}$

$\Rightarrow D$

---

3. Trigonometry, 2ADV T2 2017 HSC 7 MC

$$\begin{aligned}\tan\theta + \cot\theta &= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} \\ &= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta} \\ &= \frac{1}{\cos\theta\sin\theta} \\ &= \sec\theta \ \operatorname{cosec}\ \theta\end{aligned}$$

$\Rightarrow B$

Worked Solutions

4. Trigonometry, 2ADV T3 SM-Bank 4 MC

Period $= \dfrac{\pi}{2}$

$\Rightarrow$ must be C or D

Shift $y = \tan(x)$ right $\dfrac{\pi}{4}$.

$\Rightarrow C$

---

5. Trigonometry, 2ADV T3 SM-Bank 5 MC

Centre line (median): $y = 5$

Amplitude $= 5$

Period: $14 = \dfrac{2\pi}{n}$

$n = \dfrac{\pi}{7}$

$\therefore$ Graph: $y = 5 - 5\cos\left(\dfrac{\pi t}{7}\right)$

$\Rightarrow B$

---

6. Trigonometry, 2ADV T3 2019 HSC 7 MC

$a = \dfrac{1}{2}(5.5 - 2.5) = 1.5$

Since graph passes through $\left(\dfrac{\pi}{4}, 5.5\right)$:

$5.5 = 1.5\sin\left(b \times \dfrac{\pi}{4}\right) + 4$

$\sin\left(b \times \dfrac{\pi}{4}\right) = 1$

$b \times \dfrac{\pi}{4} = \dfrac{\pi}{2}$

$\therefore b = 2$

$\Rightarrow D$

---

7. Trigonometry, 2ADV T3 2013 HSC 6 MC

♦♦ *Mean mark 34%*

$\text{At } x = 0, \ y = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\Rightarrow$ It cannot be A or C

Find x when $y = 0$,

$$\sin\left(2x + \frac{\pi}{3}\right) = 0$$

$$\therefore \ 2x + \frac{\pi}{3} = 0 \qquad (\sin 0 = 0)$$

$$2x = -\frac{\pi}{3}$$

$$x = -\frac{\pi}{6}$$

$\Rightarrow D$

---

8. Trigonometry, 2ADV T3 2016 HSC 6 MC

♦ *Mean mark 42%.*

$$\text{Period} = \frac{\pi}{n}$$

$$= \frac{\pi}{3}$$

$\Rightarrow A$

---

9. Trigonometry, 2ADV T2 EQ-Bank 1 MC

$\tan\theta = \sqrt{3} \ \rightarrow 2$ solutions

$\cos^2\theta = 1$

$\cos\theta = \pm 1$

$\theta = 0^\circ, 180^\circ, 360^\circ \ \rightarrow 3$ solutions

$\Rightarrow C$

---

10. Trigonometry, 2ADV T2 2016 HSC 8 MC

♦♦♦ *Mean mark 23%.*

$|\cos(2x)| = 1$

$\cos(2x) = \pm 1$

When $\cos(2x) = 1$

$$2x = 0, 2\pi, 4\pi, \ldots$$

$$\therefore x = 0, \pi, 2\pi, \ldots$$

When $\cos(2x) = -1$

$$2x = \pi, 3\pi, 5\pi, \ldots$$

$$\therefore x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

$\therefore x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi \quad \text{for} \quad 0 \leq x \leq 2\pi$

$\Rightarrow D$

---

11. Trigonometry, 2ADV T2 EQ-Bank 5

$$\sin x - \cos x = 0$$

$$\frac{\sin x}{\cos x} - \frac{\cos x}{\cos x} = 0$$

$$\tan x - 1 = 0$$

$$\tan x = 1$$

$$x = \tan^{-1}(1)$$

$\therefore x = \frac{\pi}{4}, -\frac{3\pi}{4}$

---

12. Trigonometry, 2ADV T2 EQ-Bank 7

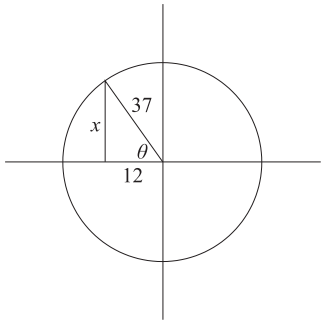
$$
\begin{aligned}
\text{LHS} &= \frac{1+\dfrac{\cos\theta}{\sin\theta}}{1+\dfrac{\sin\theta}{\cos\theta}} \\
&= \frac{\dfrac{\sin\theta+\cos\theta}{\sin\theta}}{\dfrac{\cos\theta+\sin\theta}{\cos\theta}} \\
&= \frac{\sin\theta+\cos\theta}{\sin\theta} \times \frac{\cos\theta}{\cos\theta+\sin\theta} \\
&= \frac{\cos\theta}{\sin\theta} \\
&= \cot\theta \\
&= \text{RHS}
\end{aligned}
$$

---

13. Trigonometry, 2ADV T2 SM-Bank 33

$\sec\,\theta = -\dfrac{37}{12} \Rightarrow \cos\,\theta = -\dfrac{12}{37}$

Graphically:



$x = \sqrt{37^2 - 12^2} = 35$

$$
\begin{aligned}
\therefore\ \text{cosec}\,\theta &= \frac{1}{\sin\theta} \\
&= \frac{1}{\frac{35}{37}} \\
&= \frac{37}{35}
\end{aligned}
$$

---

14. Trigonometry, 2ADV T3 2021 HSC 20

Find x such that:

$$
\begin{aligned}
2\sin 4x &= 1 \\
\sin 4x &= \frac{1}{2} \\
4x &= \sin^{-1}\frac{1}{2} \\
4x &= \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \cdots \\
\therefore x &= \frac{\pi}{24}, \frac{5\pi}{24} \quad \left(0 \le x \le \frac{\pi}{4}\right)
\end{aligned}
$$

---

15. Trigonometry, 2ADV T3 2022 HSC 14

$\text{Amplitude} = 4$

$\Rightarrow k = 4$

$\text{Period} = 6\pi$

$\dfrac{2\pi}{a} = 6\pi$

$6\pi a = 2\pi$

$\Rightarrow a = \dfrac{1}{3}$

---

16. Trigonometry, 2ADV T3 SM-Bank 12

$-1 \leq \cos\left(\dfrac{\pi x}{2}\right) \leq 1$

$-3 \leq 3\cos\left(\dfrac{\pi x}{2}\right) \leq 3$

$1 \leq 4 + 3\cos\left(\dfrac{\pi x}{2}\right) \leq 7$

$\therefore$ Range: $1 \leq y \leq 7$

$\text{Period} = \dfrac{2\pi}{n} = \dfrac{2\pi}{\frac{\pi}{2}} = 4$

---

17. Trigonometry, 2ADV T2 2023 HSC 20

$\sin 60^\circ = \dfrac{\sqrt{3}}{2} \quad \Rightarrow \quad \text{Base angle} = 60^\circ$

$\Rightarrow$ sin is negative in 3rd and 4th quadrants

$\sin(\theta - 60^\circ) = 180 + 60, 360 - 60$

$= 240^\circ, 300^\circ$

$\theta - 60^\circ = 240^\circ \quad \Rightarrow \quad \theta = 300^\circ$

$\theta - 60^\circ = 300^\circ \quad \Rightarrow \quad \theta = 360^\circ$

Consider $\theta = 0^\circ$

$\sin(0 - 60^\circ) = \sin(-60^\circ) = -\dfrac{\sqrt{3}}{2}$

$\therefore \theta = 0^\circ, 300^\circ \text{ and } 360^\circ$

---

18. Trigonometry, 2ADV T3 2018 HSC 15a

i. $L(t) = 12 + 2\cos\left(\dfrac{2\pi t}{366}\right)$

On 21 Dec 2015 $\Rightarrow t = 0$

$\therefore L(0) = 12 + 2\cos 0$

$= 14$ hours

ii. Shortest length of daylight occurs when

$\cos\left(\dfrac{2\pi t}{366}\right) = -1$

♦ *Mean mark 43%.*

$\therefore$ Shortest length $= 12 + 2(-1)$

$= 10$ hours

iii. Find t such that $L(t) = 11$:

$11 = 12 + 2\cos\left(\dfrac{2\pi t}{366}\right)$

$\cos\left(\dfrac{2\pi t}{366}\right) = -\dfrac{1}{2}$

$\dfrac{2\pi t}{366} = \dfrac{2\pi}{3}$ or $\dfrac{2\pi t}{366} = \dfrac{4\pi}{3}$

$t = \dfrac{366}{3}$ $\qquad$ $t = \dfrac{366 \times 2}{3}$

$= 122$ $\qquad$ $= 244$

$\therefore t = 122$ or 244

19. Trigonometry, 2ADV T3 2013 HSC 13a

i. $P(t) = 400 + 50\cos\left(\dfrac{\pi}{6}t\right)$

Need to find t when $P(t) = 375$

$375 = 400 + 50\cos\left(\dfrac{\pi}{6}t\right)$

$50\cos\left(\dfrac{\pi}{6}t\right) = -25$

$\cos\left(\dfrac{\pi}{6}t\right) = -\dfrac{1}{2}$

Since $\cos\left(\dfrac{\pi}{3}\right) = \dfrac{1}{2}$, and cos is negative in $2^{\text{nd}}/3^{\text{rd}}$ quadrants:

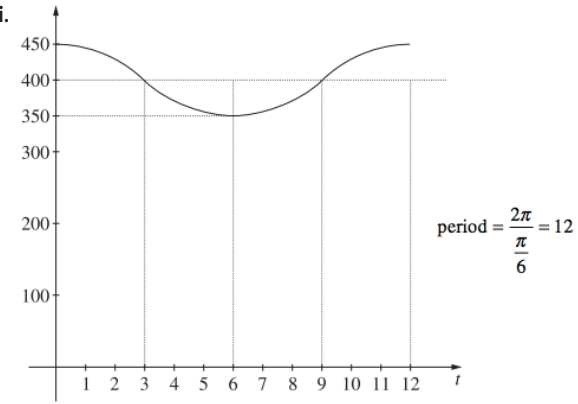
$\Rightarrow \dfrac{\pi}{6}t = \left(\pi - \dfrac{\pi}{3}\right),\ \left(\pi + \dfrac{\pi}{3}\right),\ \left(3\pi - \dfrac{\pi}{3}\right)$

$= \dfrac{2\pi}{3},\ \dfrac{4\pi}{3},\ \dfrac{8\pi}{3},\ \ldots$

$\therefore t = 4,\ 8,\ 16,\ \ldots$

$\therefore$ In the 1st 12 months, $P(t) = 375$ when $t = 4$ months and 8 months.

ii.



♦ *Mean mark 39%*

20. Trigonometry, 2ADV T3 2024 HSC 28

a. $c = \text{ centre of motion } = \dfrac{39+3}{2} = 21$

$k = \text{ amplitude } = \dfrac{39-3}{2} = 18$

b. $A(t) = 21 - 18\cos\left(\dfrac{\pi t}{24}\right) \Rightarrow n = \dfrac{\pi}{24}$

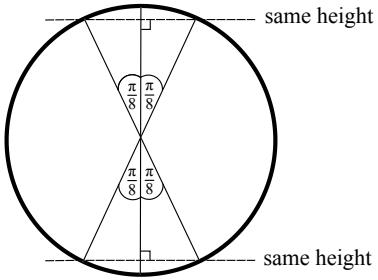
$T = \dfrac{2\pi}{n} = 2\pi \times \dfrac{24}{\pi} = 48 \text{ seconds}$

c. Billie's carriage is 6 seconds behind Anna's.

♦♦ Mean mark (c) 38%.

Angle between the 2 carriages $= \dfrac{\pi \times 6}{24} = \dfrac{\pi}{4}$

By inspection:



Heights are the same:

$h_1 = 21 - 18\cos\left(\dfrac{\pi}{8}\right) = 4.370\ldots = 4.37 \text{ m (2 d.p.)}$

$h_2 = 21 - 18\cos\left(\dfrac{7\pi}{8}\right) = 37.629\ldots = 37.63 \text{ m (2 d.p.)}$

21. Trigonometry, 2ADV T2 2014 HSC 15a

♦ *Mean mark 42%*

$$
\begin{aligned}
2\sin^2 x + \cos x - 2 &= 0 \\
2\left(1 - \cos^2 x\right) + \cos x - 2 &= 0 \\
2 - 2\cos^2 x + \cos x - 2 &= 0 \\
-2\cos^2 x + \cos x &= 0 \\
\cos x(-2\cos x + 1) &= 0
\end{aligned}
$$

$\therefore -2\cos x + 1 = 0 \quad \text{or} \quad \cos x = 0$

$2\cos x = 1 \qquad\qquad x = \dfrac{\pi}{2},\ \dfrac{3\pi}{2}$

$\cos x = \dfrac{1}{2}$

$\cos\left(\dfrac{\pi}{3}\right) = \dfrac{1}{2}$

Since cos is positive in $1^{\text{st}}/4^{\text{th}}$ quadrants,

$x = \dfrac{\pi}{3},\ 2\pi - \dfrac{\pi}{3}$

$= \dfrac{\pi}{3},\ \dfrac{5\pi}{3}$

$\therefore x = \dfrac{\pi}{3},\ \dfrac{\pi}{2},\ \dfrac{3\pi}{2},\ \dfrac{5\pi}{3} \quad \text{for} \quad 0 \le x \le 2\pi$

22. Trigonometry, 2ADV T2 2008 HSC 6a

$$2\sin^2\left(\frac{x}{3}\right) = 1 \quad \text{for} \quad -\pi \leq x \leq \pi$$

$$\sin^2\left(\frac{x}{3}\right) = \frac{1}{2}$$

$$\sin\left(\frac{x}{3}\right) = \pm\frac{1}{\sqrt{2}}$$

♦♦ *Although exact data not available, markers specifically mentioned this question was poorly answered.*

When $\sin\left(\frac{x}{3}\right) = \frac{1}{\sqrt{2}}$

$$\frac{x}{3} = \frac{\pi}{4}, \frac{3\pi}{4}$$

$$x = \frac{3\pi}{4}, \frac{9\pi}{4}$$

MARKER'S COMMENT: Many students had problems adjusting their answer to the given domain, especially when dealing with negative angles.

When $\sin\left(\frac{x}{3}\right) = -\frac{1}{\sqrt{2}}$

$$\frac{x}{3} = -\frac{\pi}{4}, -\frac{3\pi}{4}$$

$$x = -\frac{3\pi}{4}, \ -\frac{9\pi}{4}$$

$$\therefore \ x = -\frac{3\pi}{4} \ \text{ or } \ \frac{3\pi}{4} \ \text{ for } \ -\pi \leq x \leq \pi$$

---

23. Trigonometry, 2ADV T2 2019 HSC 13a

$$2\sin x\cos x - \sin x = 0$$

$$\sin x(2\cos x - 1) = 0$$

$$\sin x = 0$$

$$\Rightarrow x = 0, \ \pi, \ 2\pi$$

$$\cos x = \frac{1}{2}$$

$$\Rightarrow x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\therefore x = 0, \ \frac{\pi}{3}, \ \pi, \ \frac{5\pi}{3}, \ 2\pi$$

♦ *Mean mark 49%.*

---

24. Trigonometry, 2ADV T2 EQ-Bank 4

Prove: $\dfrac{\text{cosec}\theta + \sec\theta}{1 + \tan\theta} = \text{cosec}\theta$

$$\text{LHS} = \frac{\text{cosec}\theta + \sec\theta}{1 + \tan\theta}$$

$$= \frac{\dfrac{1}{\sin\theta} + \dfrac{1}{\cos\theta}}{1 + \dfrac{\sin\theta}{\cos\theta}} \times \frac{\cos\theta}{\cos\theta}$$

$$= \frac{\dfrac{\cos\theta}{\sin\theta} + 1}{\cos\theta + \sin\theta}$$

$$= \frac{\dfrac{\cos\theta + \sin\theta}{\sin\theta}}{\cos\theta + \sin\theta}$$

$$= \frac{1}{\sin\theta}$$

$$= \text{cosec}\theta \quad \ldots \text{as required.}$$

---

25. Trigonometry, 2ADV T3 2020 HSC 31

a. $b = \dfrac{35\ 000 + 5000}{2}$

$= 20\ 000$

$a =$ amplitude of sin graph

$= 35\ 000 – 20\ 000$

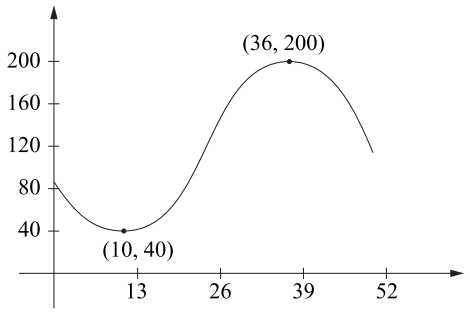
$= 15\ 000$

b. By inspection of the $m(t)$ graph

$m'(t) > 0$ when $0 \leq t < 13$ and $39 < t \leq 52$

♦♦ *Mean mark part (b) 30%.*

Sketch $c(t)$:



Minimum $(\cos 0)$ when $t = 10$

Maximum $(\cos \pi)$ when $t = 36$

$\therefore c'(t) > 0$ when $10 < t < 36$

$\therefore$ Both populations are increasing when $10 < t < 13$

c. $c(t)$ maximum when $t = 36$

$m(t) = 15\ 000 \sin\left(\dfrac{\pi}{26}t\right) + 20\ 000$

♦♦♦ *Mean mark part (c) 27%.*

$m'(t) = \dfrac{15\ 000\pi}{26} \cos\left(\dfrac{\pi}{26}t\right)$

$m'(36) = \dfrac{15\ 000\pi}{26} \cdot \cos\left(\dfrac{36\pi}{26}\right)$

$= -642.7$

$\therefore$ Mice population is decreasing at 643 mice per week.

Copyright © 2016-2025 M2 Mathematics Pty Ltd (SmarterMaths.com.au)